Evaluation of line-wise and semi region-wise phase unwrapping for isoclinic parameter in digital photoelasticity

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ABSTRACT

Isoclinic parameter is one of the important parameters in (digital) photoelasticity. This paper presents a comparative study of two phase unwrapping methods for the parameter. The two phase unwrapping methods are a line-wise method and a semi-region-wise method. The paper focuses on the way that how the methods handle the wrapped isoclinic map(s), which is used to start unwrapping, having the edge-cut line and non-edge-cut line in the final or unwrapped isoclinic map. The edge-cut line and non-edge-cut line are the line of isoclinic jumps that link the points of discontinuity (isotropic points) with some points locating at the boundaries and inside the isoclinic map, respectively. For the evaluation, three models, the circular disk and ring under diametral compression and circular disk under three radial loads, were used. The comparative results show that the semi region-wise method is superior to the line wise method.

Keywords: Digital photoelasticity, isoclinic parameter, line-wise phase unwrapping, semi region-wise phase unwrapping

1. INTRODUCTION

Determination of the isoclinic parameter or the principal stress directions is very important in the field of (digital) photoelasticity since the isoclinic parameter has to be used in stress separation, particularly, the shear difference technique. The problem in determination of this parameter occurs, even though the equation used for the calculation is theoretically derived. This equation can be found in any text book of Mechanics of Materials and is expressed as

$$\tan 2\phi = \frac{2\tau_{xy}}{\sigma_{xx} - \sigma_{yy}}, \tag{1}$$

where ϕ is the isoclinic parameter, σ_{xx}, σ_{yy}, and τ_{xy} are the Cartesian normal and shear stress components determined point-wise. It is seen from Eq. (1) that ϕ lies in the range of $-\pi/4$ to $+\pi/4$ when using the ordinary atan function (ATAN) or of $-\pi/2$ to $+\pi/2$ when using the sign dependent atan function (ATAN2) for all over the domain. It is to be noted that the physical range of the isoclinic parameter is of $-\pi/2$ to $+\pi/2$. That the determined range of isoclinic parameter is less than the physical range is known as wrapped isoclinics. One may go to a wrong conclusion that use of ATAN2 readily solves the wrapped problem; however, this is not true since there are only some regions representing the isoclinic parameter in the physical range while other regions show other wrapped ranges. This situation makes inconsistent distribution of isoclinic parameter over the entire domain.

Returning to the use of ATAN, the isoclinic parameter lies in the range of $-\pi/4$ to $+\pi/4$; hence, a phase unwrapping is needed to bring the isoclinic parameter to the physical range. Several phase unwrapping techniques are provided and one of them is typically used, i.e., the line-wise phase unwrapping. The author and his co-worker have already developed a phase unwrapping technique [1] and this technique can handle complex problems. According to its principle, it is named as a semi region-wise phase unwrapping since it starts unwrapping from one largest region but performs unwrapping pixel-wise in the mask window. To see the performance of these two phase unwrapping techniques, this paper focuses on a comparative evaluation of the results obtained from them when they are applied to several photoelastic models, i.e., the circular disk and ring under diametral compression and a circular disk under three radial loads.

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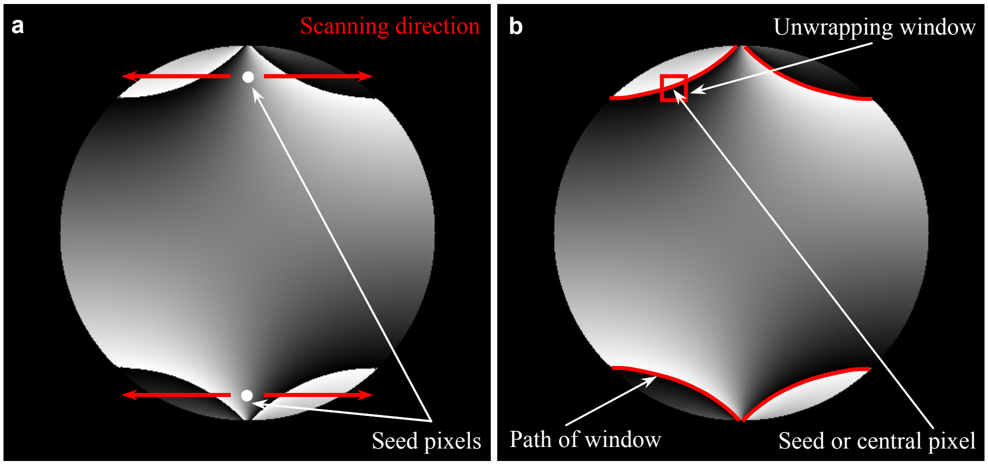

Fig. 1. Examples of unwrapping procedures of line-wise and semi region-wise techniques starting from the wrapped isoclinic map in the range of $-\pi/4$ to $+\pi/4$ for a circular disk model under diametral compression. (a) line-wise technique with the horizontal scanning and (b) semi region-wise technique with a 3 × 3-pixel window moving on a boundary or a path. Note that the size of window is exaggerated for the reason of clarity.

2. ISOCLINIC UNWRAPPING TECNIQUES

To clearly see their performance, it is important to understand the principles of the two phase unwrapping techniques. Then, in this section, their principles are briefly described.

2.1 Line-wise technique [2-5]

The principle of the line-wise technique is to unwrap a wrapped pixel row-wise or column-wise (Fig. 1a). This can be achieved by scanning the pixel along such directions form the seed pixels. The process of unwrapping is done on the basis of the comparison between the isoclinic values of two successive considered pixels. Note that for the seed pixel, it is considered to be already unwrapped and, typically, manually determined by the analyst.

This technique starts unwrapping process with the use of the isoclinic parameter in the range of $-\pi/4$ to $+\pi/4$ (Fig. 1). As mentioned, the seed pixels are given by the analyst and the unwrapping process is done horizontally. Note that this can be done vertically depending on the position of the seed pixel. The resultant isoclinic value of the pixel being considered is obtained by $-\pi/2$ or $+\pi/2$ phase shift to the wrapped isoclinic value of that pixel with a predefined condition; that is, if the difference between the isoclinic values of the seed pixel and the considered pixel is slightly less than $\pi/2$ (generally, $\pi/3$), the phase shift operation is activated. The direction of the phase shift depends on the continuity of the unwrapped isoclinic value. Referring to Fig.1a, this process is done for all four small regions.

2.2 Semi region-wise technique [1]

The principle of the semi region-wise technique is almost similar to the line-wise technique. However, the main difference is the way that it automatically determines the major and minor seed pixels and the region for starting unwrapping process. Note that the phase shift operation is totally the same but the unwrapping direction is unnecessary to be only horizontal or vertical since the wrapped pixels only inside the window are treated at a time. When unwrapping, a 3 × 3-pixel window (Fig. 1b) is randomly moved along the boundary (path) of the unwrapped region used to start the process with a controlling variable - a deviation between the unwrapped isoclinic values and their averaged value of all pixels on the boundary.

It is to note that the semi region-wise technique can deal with fringe fields of various photoelastic models, regardless the presence of the discontinuities in such fringe fields since it has a sub procedure to identify the discontinuities such that they are lastly processed. Apart from the main difference mentioned above, the semi region-wise technique requires two ranges of isoclinic parameter, i.e., 0 to $+\pi/2$ and $-\pi/4$ to $+\pi/4$, instead of only one range as needed by the line-wise technique. It will be seen, however, later about the usefulness of the isoclinics in the range of 0 to $+\pi/2$.

3. RESULTS AND COMPARATIVE EVALUATION

This section presents the obtained results for all models studied as well as the comparative evaluation between the results obtained from the two techniques is addressed. To save the space of this paper, the raw photoelastic fringes of those models are not shown. For visualization, the isoclinic values are changed to image by mapping into the 255-gray scale with following equation.

$$\phi_g = \mathrm{INT}\left[\frac{255}{\Delta\phi}(\phi_u - \phi_{min})\right], \quad (2)$$

where ϕ_g is the pixel intensity of the map image, ϕ_u is true unwrapped values of isoclinics, $\Delta\phi = \phi_{max} - \phi_{min}$, ϕ_{max} and ϕ_{min} are the maximum and minimum values of ϕ_u obtained from the unwrapping process, and INT is the function giving a nearest integer value.

3.1 Circular disk under diametral compression

After applying those two unwrapping techniques to the map of isoclinic parameter in the range of $-\pi/4$ to $+\pi/4$ for the line-wise method and the ranges of 0 to $+\pi/2$ and $-\pi/4$ to $+\pi/4$ for the semi region-wise method, the unwrapped maps were given using Eq. (2) and Fig. 2 shows these unwrapped maps.

Qualitative inspection of Fig. 2 shows that both maps are almost similar (see the small difference at the bottom part). Note that the unwrapped map of isoclinic parameter represents the σ_1 direction over the entire domain. Further observation reveals that there exists the effect of the isochromatic parameter on the unwrapped map and it appears as an incomplete ellipse. This isochromatic parameter is of the first order ($N = 1$, where N is the fringe order). The fringe order was found to be the first by comparing the colors of photoelastic fringe at that position with those reported in reference [6]. It is to note that Fig. 2a was obtained using the technique shown in Fig. 1a whereas Fig. 2b was given from the use of technique shown in Fig. 1b.

For the quantitative comparison, the distribution of the isoclinic values selected along the (red) line shown in Fig. 2a shows that the isoclinic values of Figs. 2a and 2b are identical and the graph is not shown here.

3.2 Circular ring under diametral compression

Figure 2 shows the unwrapped maps obtained using the line-wise method (Fig. 2a) and the semi region-wise method (Fig.2b). It is seen that, for the most parts of the maps, they look similar. The different parts are at and near the isotropic points and the load application points. At the load application point (top and bottom points), the differences may be due to the small deformation and the imperfection of the model.

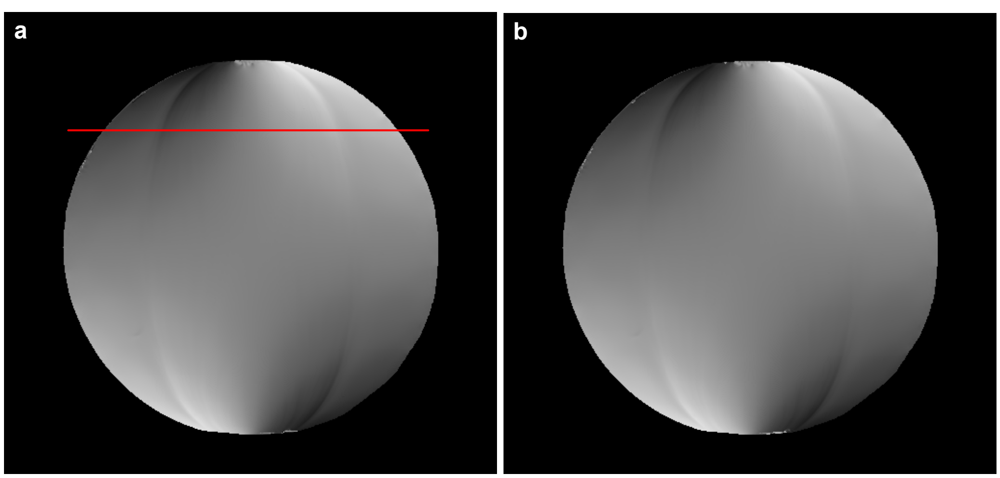

Fig. 2. Unwrapped map of isoclinic parameter in the range $-\pi/2$ to $+\pi/2$ of the circular disk obtained using (a) line-wise technique with a horizontal scanning and (b) semi region-wise technique. These maps represent the σ_1 direction and are digitally represented by mapping the minimum and maximum isoclinic values to black and white, respectively.

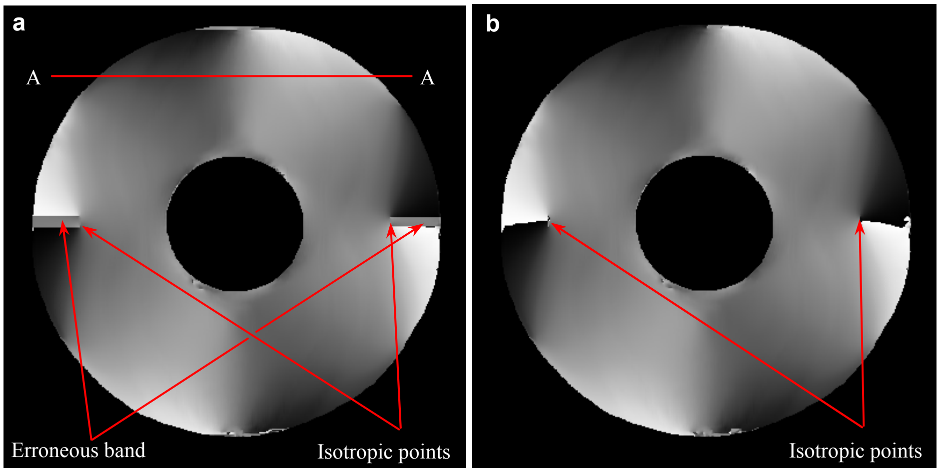

Fig. 3. Unwrapped map of isoclinic parameter in the range −π/2 to +π/2 of the circular ring obtained using (a) line-wise technique with a horizontal scanning and (b) semi region-wise technique. These maps represent the σ_1 direction and are digitally represented by mapping the minimum and maximum isoclinic values to black and white, respectively.

At the isotropic points of Fig. 3a, there are erroneous bands linking between the isotropic points and the points on the boundaries. Obviously, Fig. 3b shows no any erroneous bands. These bands occurred because the isotropic points represent themselves as a region. If the scanned lines pass through such region when unwrapping, the pixels next to the isotropic points possess wrong unwrapped isoclinic values and these propagate to other adjacent pixels as well. The true reason why that situation happened is that, at the isotropic point, the value of isoclinic parameter is indeterminate and the isoclinic parameter can be of any value [7]. Since, as mentioned in Sec. 2.1, the process of unwrapping is done on the basis of the comparison between the isoclinic values of two successive pixels, the difference of isoclinic values is incorrectly determined and it results such situation.

Consider the distributions of the isoclinic values along the (red) line A-A drawn on Fig. 3a. It was found that the distributions obtained from two maps are almost similar. The difference was due to the unequal values of ϕ_{max} and ϕ_{min} between the maps; that is, the values of ϕ_{max} and ϕ_{min} of Fig. 3a were slightly greater and lower than those of Fig. 3b, respectively. This made the lower distribution of isoclinics of Fig. 3a. However, since the number of pixels possessing such values of ϕ_{max} and ϕ_{min} (Fig. 3a) were not much, they could be considered to be noise. If the pixel values were forced to be in the range of −π/2 to +π/2, the final distributions were, then, identical.

3.3 Circular disk under three radial compressive loads

By the same way as done in Sec. 3.1 and Sec. 3.2, the unwrapped maps of the circular disk under three radial loads were given with Eq. (2). Figure 4 shows the unwrapped maps. Figure 4a was obtained using the technique shown in Fig. 1a whereas Fig. 4b was provided by the use of technique shown in Fig. 1b.

Qualitative inspection of Fig. 4 reveals that the maps look different. Considering first for the upper part above the isotropic point, it is obviously observed that the isoclinic values are similar except for the values at the left and right boundaries. For the left boundary, Fig. 4a shows higher reliability of isoclinic values than those of Fig. 4b whereas this situation is found to be reversed for the right side. However, the difference between the left and right sides is not significant.

For the lower part below the isotropic point, these maps show no similarity. It is seen that Fig. 4a has the edge-cut line whereas Fig. 4b has the non-edge-cut line; however, they should be the same. Upon closer scrutiny the image of isoclinic lines reported in the book's Frocht [7], it was found that the map in Fig. 4b was correct. This is because, along that line, it should have the common point of 0°isoclinic since the lines of phase jumps at 45° were melt down and the common point of 0°isoclinic was preserved. However, in Fig. 4a, the common point of 0°isoclinic was destroyed by the line-wise method. That the common point of 0°isoclinic still exists in Fig. 4b is due to the use of the isoclinics in the range of 0 to +π/2 which enforces the unwrapping algorithm to preserve that common point.

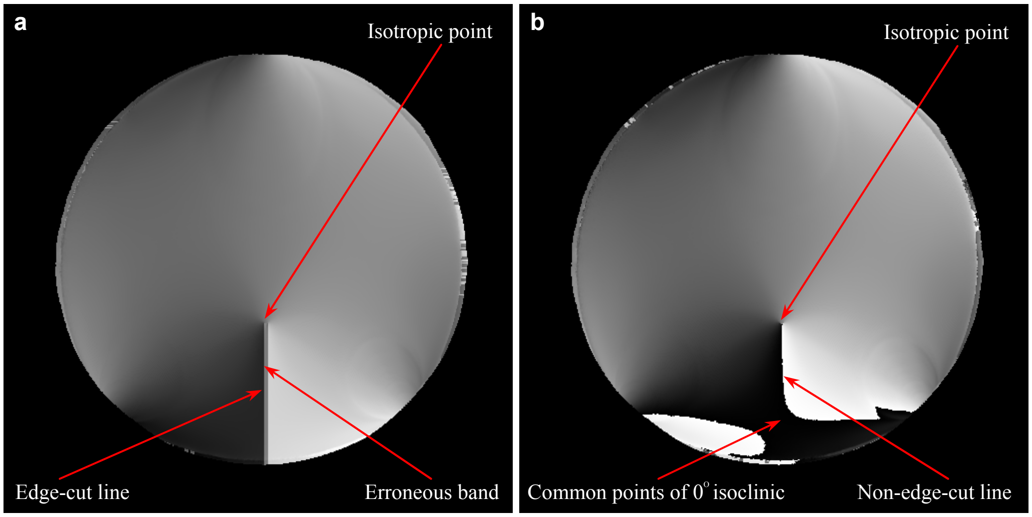

Fig. 4. Unwrapped map of isoclinic parameter in the range −π/2 to +π/2 of the circular disk under three radial loads obtained using (a) line-wise technique with a horizontal scanning and (b) semi region-wise technique. These maps represent the σ_1 direction and are digitally represented by mapping the minimum and maximum isoclinic values to black and white, respectively.

Apart from those mentioned, the edge-cut line in Fig. 4a (actually it should be the non-edge-cut line) represents itself as a small wide band instead of a line whereas this line is not seen in Fig. 4b. This band occurred because the isotropic point is actually not a point in the photoelastic fringe field but rather a region. Therefore, when the scanned line passing through the isotropic point or region, the wrong isoclinic values were obtained and these wrong values affected all other values after them along that scanned line.

It is of importance to note that if the values of ϕ_{max} and ϕ_{min} were intentionally set to +π/2 and −π/2, respectively, the unwrapped maps of isoclinics obtained from both methods were the same as that shown in Fig. 4b. This peculiar situation is due to the reciprocal order of the gray values since the values of ϕ_{max} or ϕ_{min} obtained from the line-wise method were somewhat greater than +π/2 or lower than −π/2. One may wonder why the situation is different from that of the circular ring model as explained in Sec. 3.2 in the last paragraph. The reason is that this model has the non-edge-cut line that links between the isotropic point and the common point of 0°isoclinic whereas, for the circular ring model, it has the edge-cut line. Further, the number of pixels possessing the isoclinic values greater than +π/2 or lower than −π/2 (Fig. 4a) were much more than those of Fig. 3a (more than 45%); therefore, these pixels were not the noise but they were the wrong unwrapped isoclinic values rendered form the line-wise method. As mentioned earlier, Fig. 4 has no these erroneous pixels and this results from the act of the isoclinics in the range of 0 to +π/2.

4. CONCLUSION

Two phase unwrapping methods, the line-wise and semi region-wise methods, are used to determine the unwrapped map of isoclinics and their results are compared as well. Three models, circular disk and circular ring under diametral compression and circular disk subjected to three radial loads, are used for the test of the unwrapping methods. The results form the unwrapping algorithms show a very high degree of similarity for the first two models. However, a large deviation between the unwrapped maps is obviously seen for the last model – the circular disk subjected to three radial loads. This deviation is because for the line-wise method there is no any information or criteria to notify the algorithm to do when the isoclinic values goes beyond +π/2 or −π/2 after translation of values whereas the isoclinic values in the range of 0 to +π/2 used in the semi region-wise method guide the algorithm how to do if such situation happens. As seen, the use of mapping equation (Eq. (2)) is crucial since the unwrapped isoclinic map is strongly dependent of the values of ϕ_{max} and ϕ_{min}. Using wrong values of ϕ_{max} and ϕ_{min} leads to wrong unwrapped map of isoclinics. It is to be noted that when working with the wrapped map having the edge-cut line there is no any deviation for the unwrapped maps obtained from those two methods; however, the deviation may present if the wrapped map has the non-edge-cut line and the common point of 0°isoclinic. This situation is difficult to predict and cannot be known in advance when dealing with other models. For future work, the semi region-wise method is, therefore, applied to various models in order to confirm the unwrapped maps of isoclinics.

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