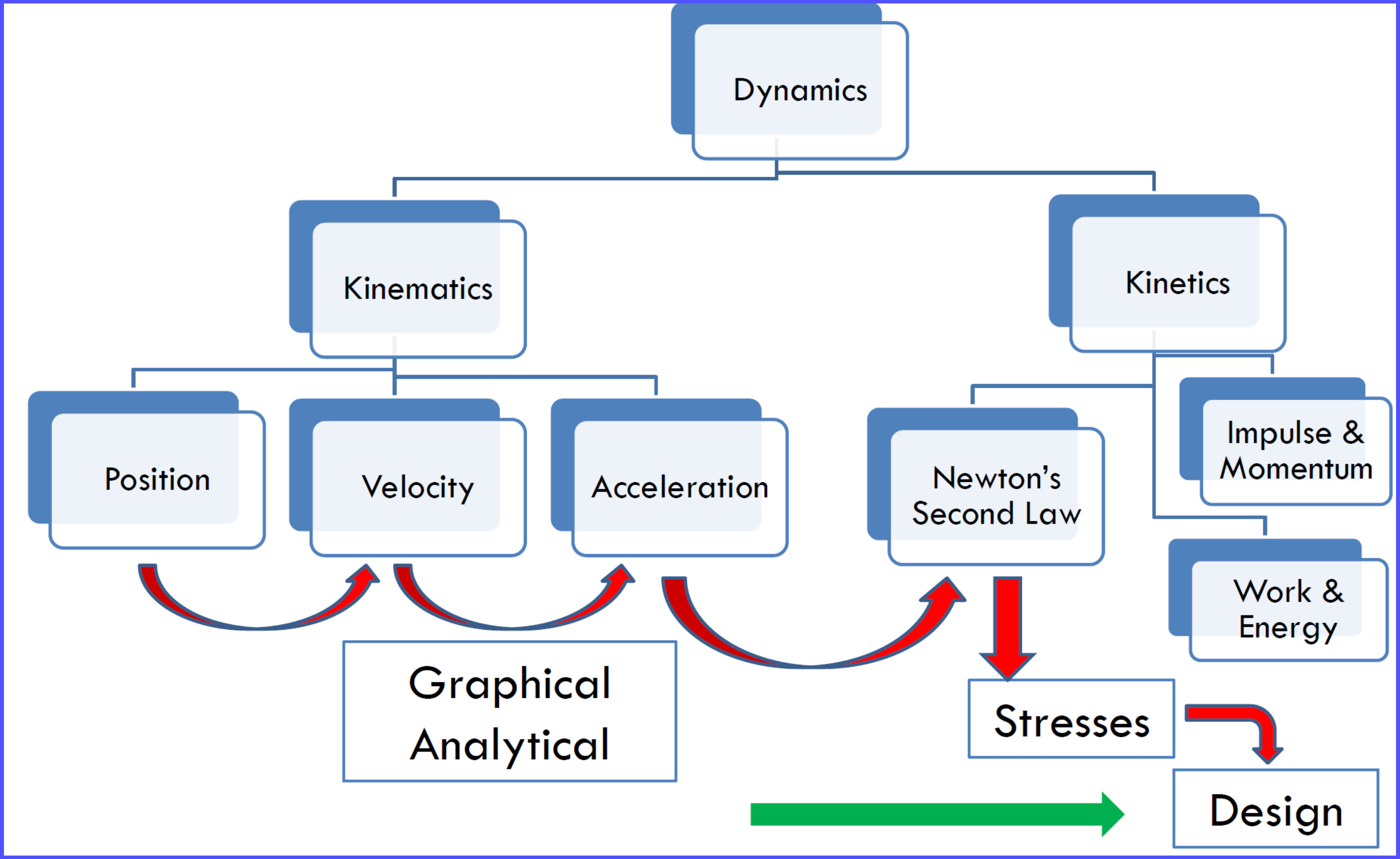
Kinematic Analysis: Scope



- Need to know the dynamic forces to be able to compute stresses in the components
- Dynamic forces are proportional to acceleration (Newton second law)
- Goal shifts to finding acceleration of all the moving parts in the assembly

- In order to calculate the accelerations:
  - need to find the positions of all the links , for all increments in input motion
  - differentiate the position eqs. to find velocities, diff. again to get accelerations

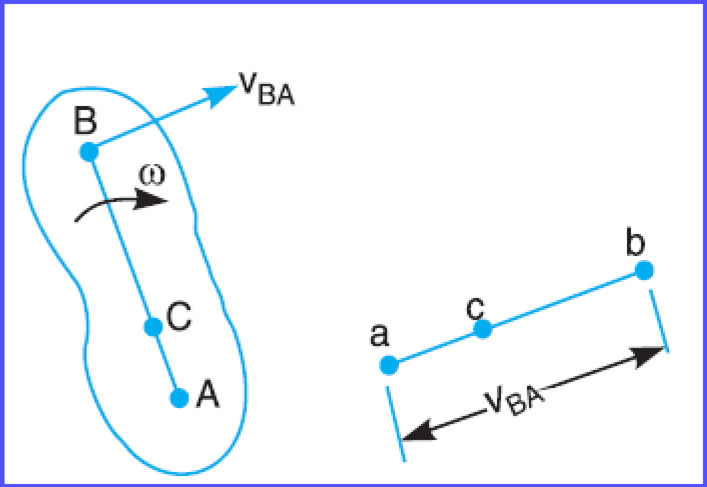
Velocity analysis: overview of methods

Velocity: Rate of change of position with respect to time

Graphical methods
Relative velocity method
Instantaneous centre method

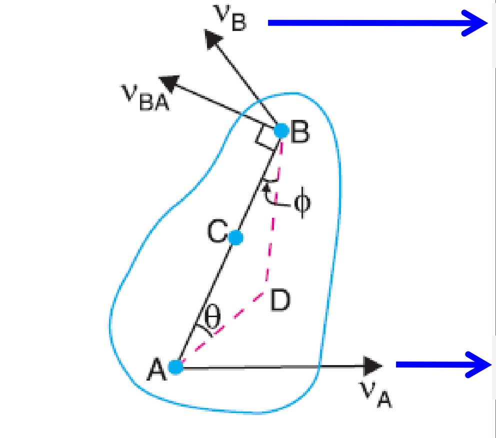
Vector loop method

Velocity of any point on a link with respect to another point on the same link
Is always perpendicular to the line joining these points on the configuration (or space) diagram



Velocity analysis: Relative velocity method

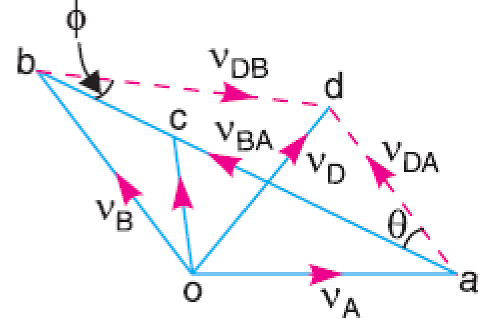
1. Take some convenient point *o*, known as the pole.
2. Through *o*, draw *oa* parallel and equal to v_A, to some suitable scale.
3. Through *a*, draw a line perpendicular to *AB*. This line will represent the velocity of *B* with respect to *A*, *i.e.* v_{BA}.
4. Through *o*, draw a line parallel to v_B intersecting the line of v_{BA} at *b*.
5. Measure *ob*, which gives the required velocity of point *B* (v_B), to the scale.



(*a*) Motion of points on a link.

Only direction known

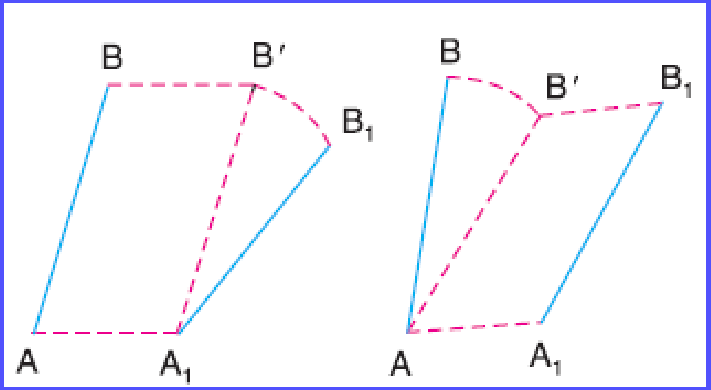
Completely known

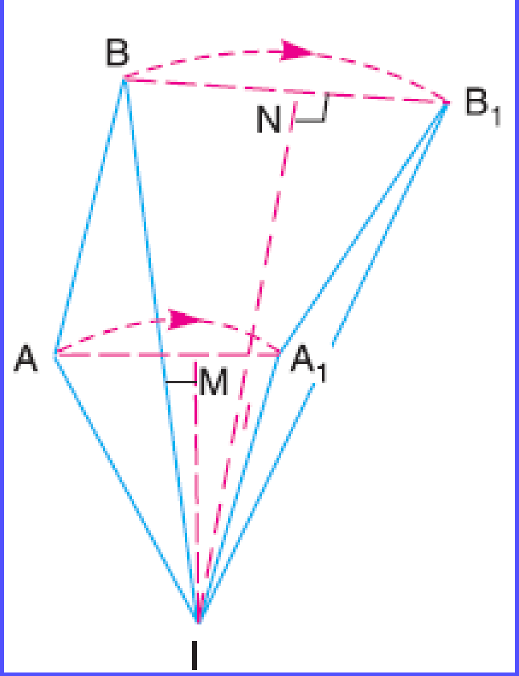


(*b*) Velocity diagram.

Velocity analysis: Instantaneous centre method

Complex Motion as a case of pure rotation





As the position of link AB goes on changing, so does the centre I, about which AB is assumed to be rotating. Hence, the name Instantaneous Centre.

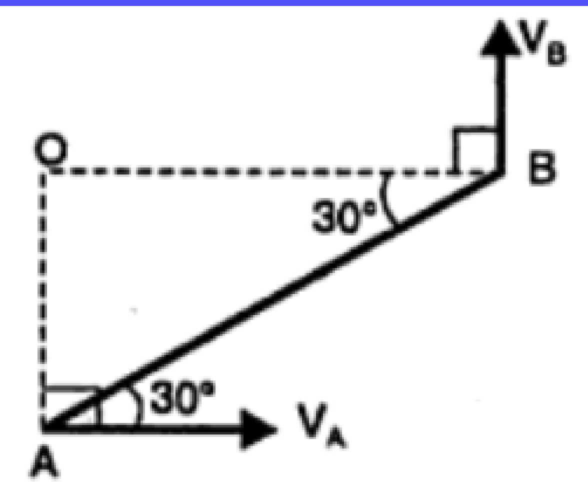
The locus of all such instantaneous centres is known as centrode.

A line drawn through an instantaneous centre and perpendicular to the plane of motion is an instantaneous axis.

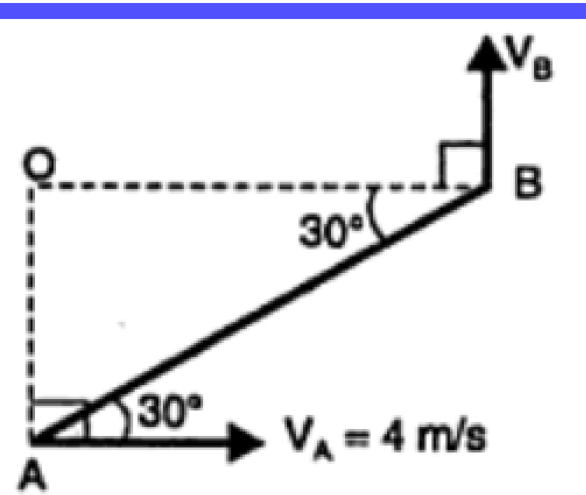
The locus of instantaneous axis is known as axode. (axis+centrode=axode)

Velocity analysis: Instantaneous centre method

Locating an Instantaneous Center of Rotation, and its use



Just two directions of velocities, help locate the IcR



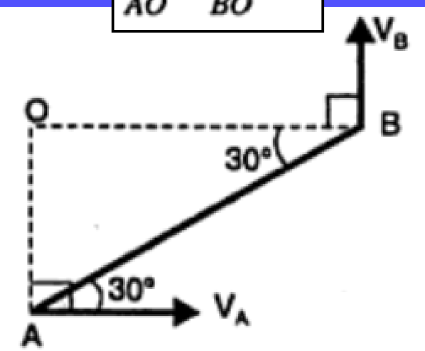
One complete velocity (magnitude + direction)
&
one other velocity direction,
helps find velocity of any other point.

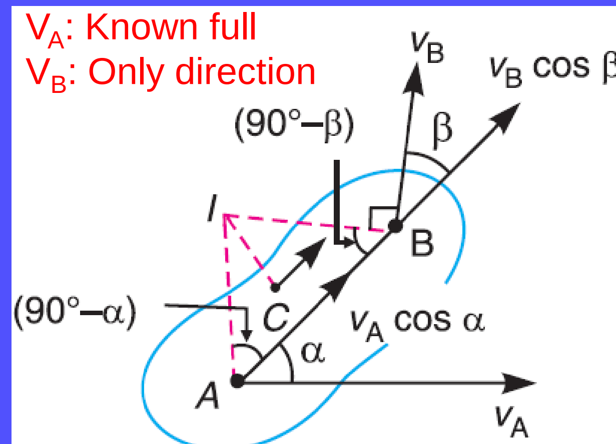
$$\frac{V_A}{AO} = \frac{V_B}{BO} = \omega \qquad \frac{V_A}{AO} = \frac{V_B}{BO} = \frac{V_C}{CO} = \omega$$

Velocity analysis: Instantaneous centre method

Locating an Instantaneous Center of Rotation, and its use

$$\frac{V_A}{AO} = \frac{V_B}{BO} = \omega$$





No relative motion between A and B

$$v_A \cos\alpha = v_B \cos\beta \Rightarrow \frac{v_A}{v_B} = \frac{\cos\beta}{\cos\alpha} = \frac{\sin(90° - \beta)}{\sin(90° - \alpha)}$$

Lami's theorem to triangle ABI ⇒

$$\frac{AI}{\sin(90° - \beta)} = \frac{BI}{\sin(90° - \alpha)} \Rightarrow \frac{AI}{BI} = \frac{\sin(90° - \beta)}{\sin(90° - \alpha)}$$

$$\frac{v_A}{v_B} = \frac{AI}{BI}$$

$$\frac{v_A}{AI} = \frac{v_B}{BI} = \omega$$

Velocity analysis: Instantaneous centre method

More on Instantaneous Centres

No of Instantaneous Centres = No. of possible combinations of two links
= No. of combinations of n links taken two at a time

$$N = \frac{n(n-1)}{2}, \text{ where } n = \text{Number of links.}$$

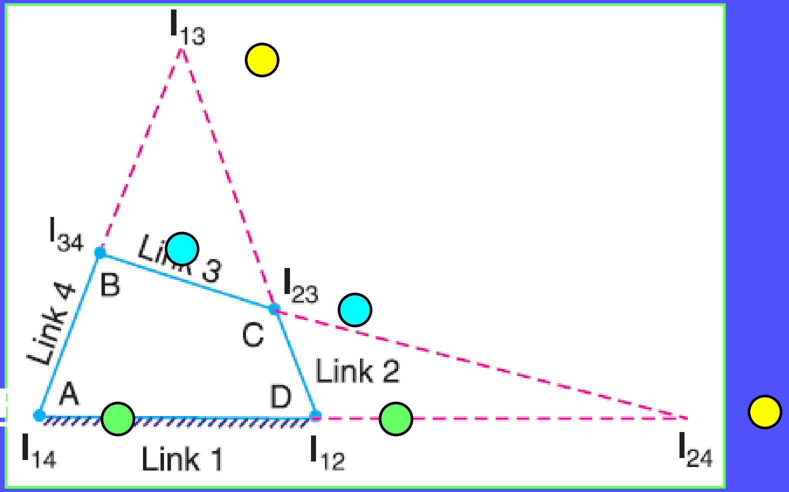
pes of Instantaneous Centres

Fixed: Remain in the same place for all configurations of the mechanism

Permanent: Change positions but the nature of joints is permanent

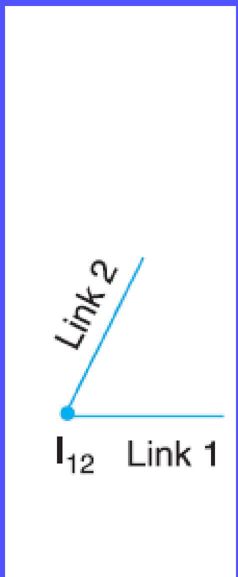
} **Primary IcR**

Neither fixed nor permanent } **Secondary IcR**

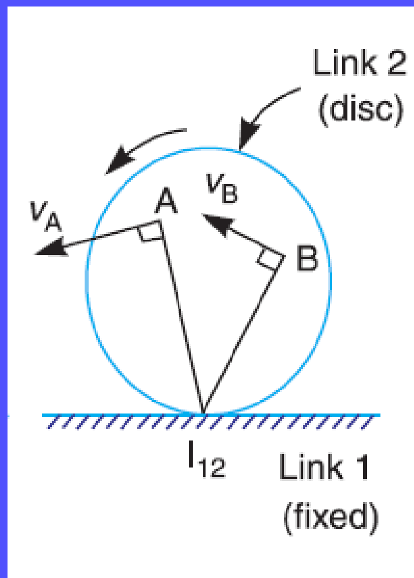


Velocity analysis: Instantaneous centre method

Rules for locating Instantaneous Centres

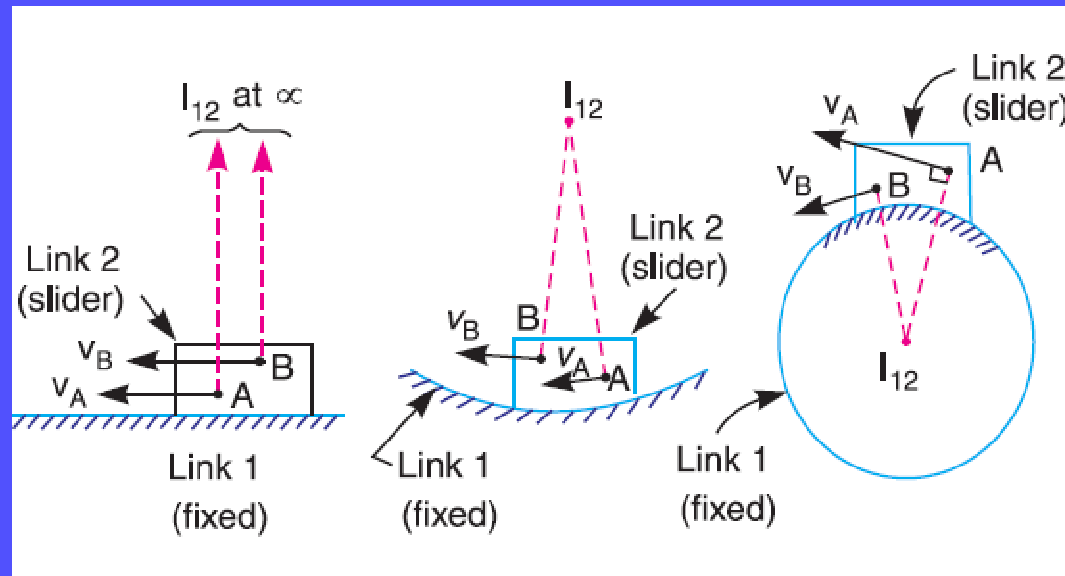


For two links connected by a **pin joint**, the IcR lies on the **centre of the pin**



When the two links have a **pure rolling** (no slipping) contact, the IcR lies **on their point of contact**

When the two links have a sliding contact, the IcR lies on the common normal at the point of contact



The ICR lies at infinity, and each point on the slider has the same velocity

The ICR lies on the centre of curvature, of the curvilinear path, at that instant

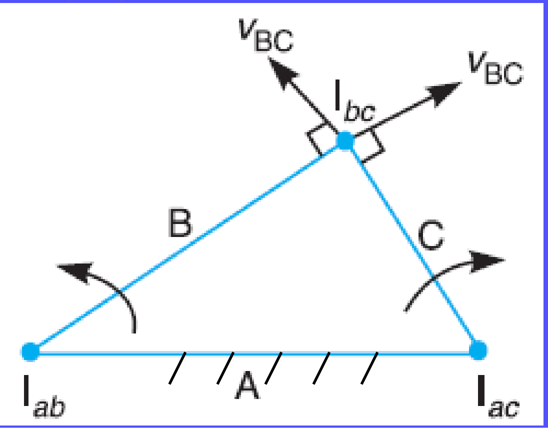
The ICR lies on the centre of curvature, which being the centre of the circle is fixed for all configurations of the links.

Velocity analysis: Instantaneous centre method

Aronhold Kennedy (or Three Centres in Line) Theorem

Three links: A, B, & C, having relative plane motion.

$$N = \frac{n(n-1)}{2} = \frac{3(3-1)}{2} = 3$$



Aronhold Kennedy's theorem: **if three bodies move relative to each other, they have three Instantaneous centres, and they lie on a straight line.**

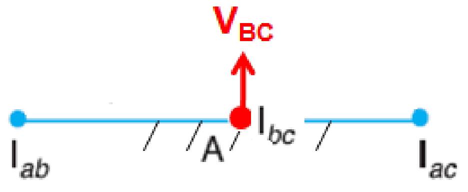
I_{bc} must lie on the line joining I_{ab} and I_{ac}

Consider I_{bc} lying outside the line joining I_{ab} and $I_{ac.}$ Now I_{bc} belongs to both the links B and C.

Consider I_{bc} ϵ link B: V_{BC} must be perpendicular to the line joining I_{ab} and $I_{bc.}$

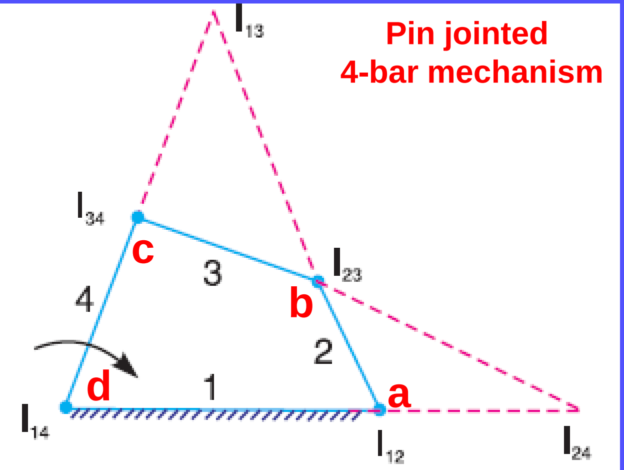
Consider I_{bc} ϵ link C: V_{BC} must be perpendicular to the line joining I_{ac} and $I_{bc.}$

But I_{bc} is a unique point; and hence, regardless of whether it ϵ link B or Link C, it should have a unique velocity (magnitude and direction). This is possible only when the three instantaneous centres, namely, $I_{ab,}$ I_{ac} and I_{bc} lie on the same straight line.



The exact location of I_{bc} on the line I_{ab} I_{ac} depends on the directions and magnitudes of the angular Velocities of B and C relative to A.

Velocity analysis: Instantaneous centre method



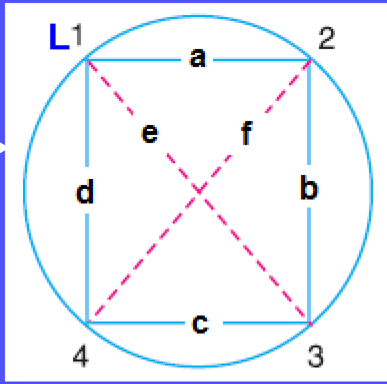
1. Determine the number of instantaneous centres (N) by using the relation

$$N = \frac{n(n-1)}{2}, \text{ where } n = \text{Number of links.}$$

2. Make a list of all the IcRs in the mechanism →

1	2	3	4
12	23	34	–
13	24		
14			

3. Locate by inspection, the primary IcRs, and mark them by solid lines, on the circle diagram → I_{12}, I_{23}, I_{34} and I_{14}.



Circle diagram

4. Locate the secondary IcRs using kennedy's theorem: **if three bodies move relative to each other, they have three Instantaneous centres, and <u>they lie on a straight line.</u>**

↓

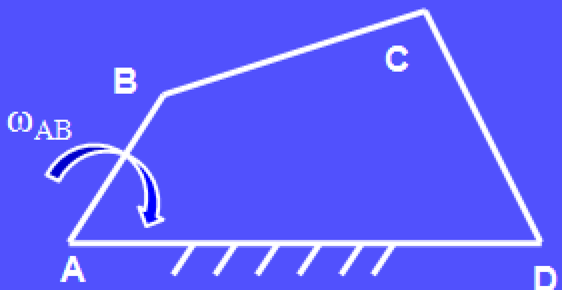
To implement KnDT: Look for quadrilaterals in the circle diagram, and form diagonals. Clearly each diagonal (say, 1-3) will form two adjacent triangles (1-3-4, and, 1-2-3), that is, **each diagonal will form 2 pairs of three bodies in relative motion, to each of which KnDT can be applied**

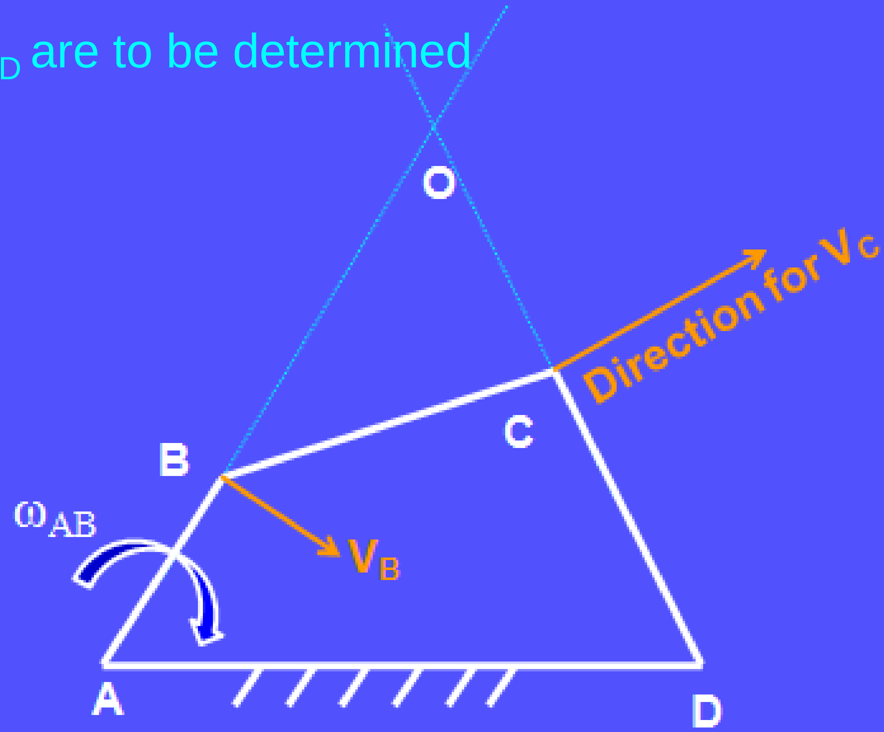
I_{13} will lie on the intersection of I_{12}-I_{23} (3 bodies: 1-2-3) and I_{14}-I_{34} (3 bodies: 1-3-4), produced, if necessary.

I_{24} will lie on the intersection of I_{12}-I_{14} (3 bodies: 1-2-4) and I_{23}-I_{34} (3 bodies: 2-3-4), produced, if necessary.

Velocity analysis: Instantaneous centre method

ω_{AB} is given, and ω_{BC} and ω_{CD} are to be determined





Step-I: Perpendiculars to the two known direction of velocities of B & C help locate the IcR at O

Step-II: Point B belongs to both:
- **the link AB, under pure rotation about A**
- **the link BC, under complex motion, equivalent to pure rotation about O.**

$$\omega_{AB} * AB = \omega_{BC} * BO$$

ω_{BC} already known by now

Step-III: Point C belongs to both:
- **the link CD, under pure rotation about D**
- **the link BC, under complex motion, equivalent to pure rotation about O.**

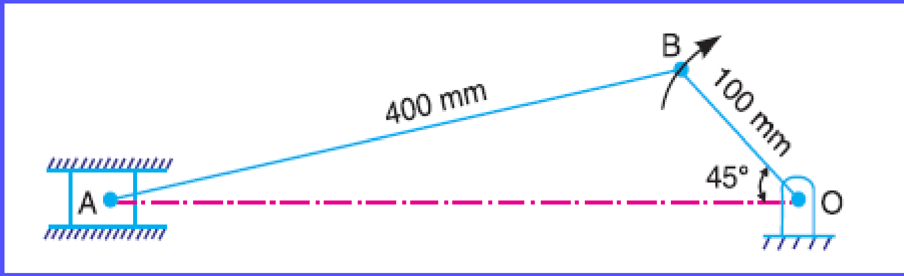
$$\omega_{CD} * CD = \omega_{BC} * CO$$

Velocity analysis: Instantaneous centre method: Exp-I

Locate all the IcRs of the slider crank mech. shown in the figure. The lengths of crank OB and connecting rod AB are 100 and 400 mm, respectively. If the crank rotates clockwise with an angular velocity of 10 rad/s, find:

(i) Velocity of the slider A, and

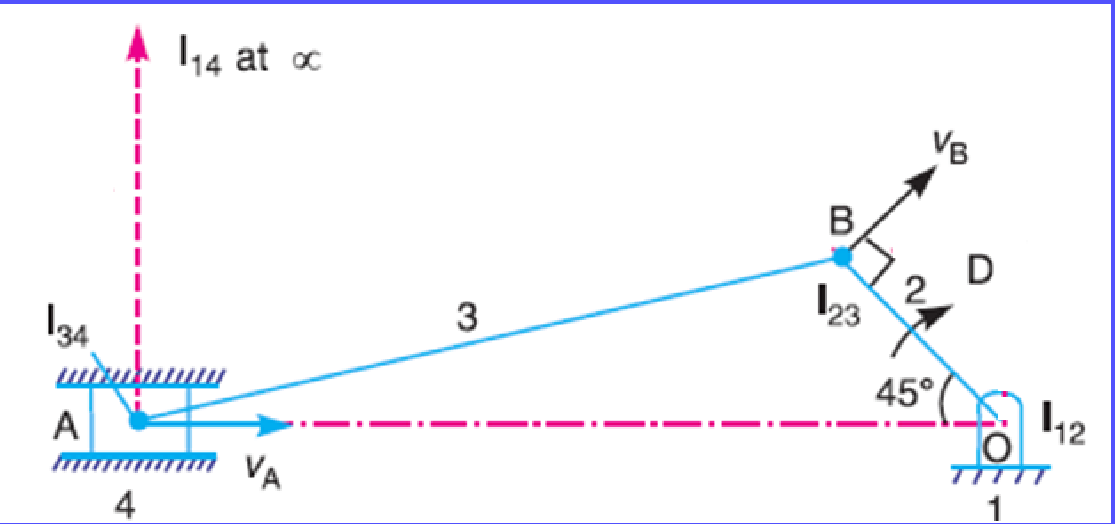
(ii) Angular velocity of the connecting rod AB.

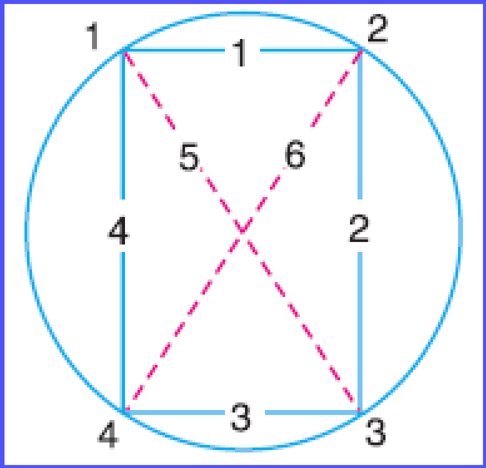


$$\omega_{OB} = 10 \text{ rad/ s}; \ OB = 100 \text{ mm} = 0.1 \text{ m}$$

$$v_{OB} = v_B = \omega_{OB} \times OB = 10 \times 0.1 = 1 \text{ m/s}$$

$$N = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$$

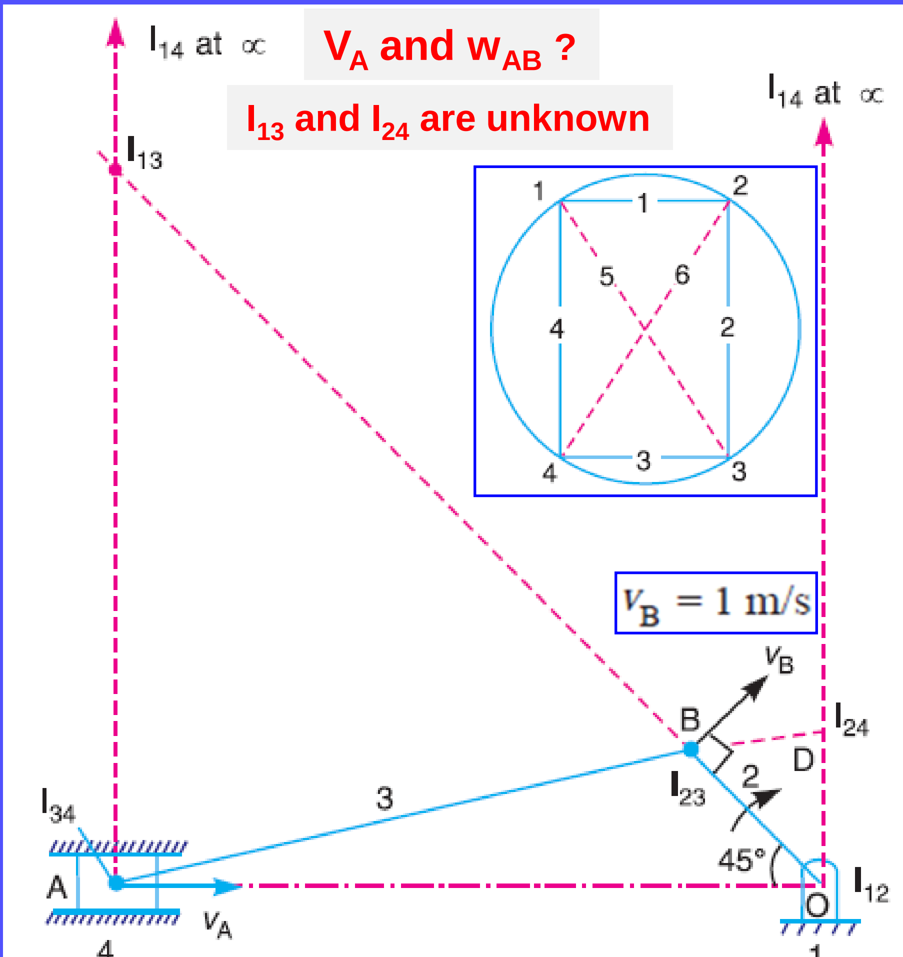




$I_{12}, I_{23}, I_{34}, I_{14}$ **known**

I_{13} and I_{24} : unknown

Velocity analysis: Instantaneous centre method: Exp-I



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I_{24} will lie on the intersection of I_{12}-I_{14} (3 bodies: 1-2-4) and I_{23}-I_{34} (3 bodies: 2-3-4), produced, if necessary.

$I_{13}A = 460 \text{ mm} = 0.46 \text{ m}$; and $I_{13}B = 560 \text{ mm} = 0.56 \text{ m}$

AB is having a complex motion, equivalent to pure rotation about I_{13}

1. *Velocity of the slider A*

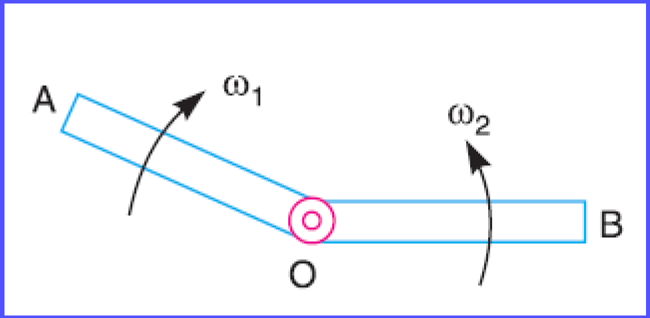
Let v_A = Velocity of the slider A.

We know that $\dfrac{v_A}{I_{13}A} = \dfrac{v_B}{I_{13}B}$ $\qquad v_A = v_B \times \dfrac{I_{13}A}{I_{13}B} = 1 \times \dfrac{0.46}{0.56} = 0.82 \text{ m/s}$

2. *Angular velocity of the connecting rod AB*

$$\omega_{AB} = \frac{v_B}{I_{13}B} = \frac{1}{0.56} = 1.78 \text{ rad/s}$$

Velocity analysis: Rubbing velocity at a pin joint



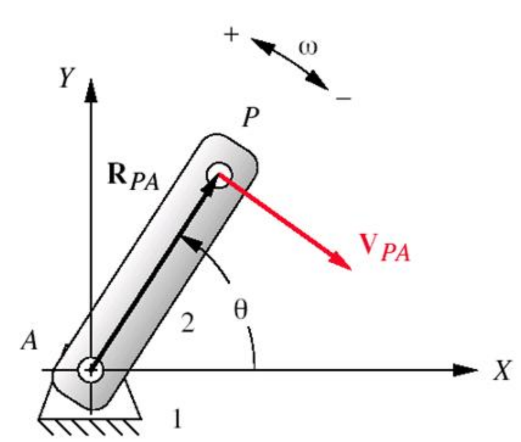
Rubbing velocity at the pin joint O

$$= (\omega_1 - \omega_2)\, r, \text{ if the links move in the same direction}$$

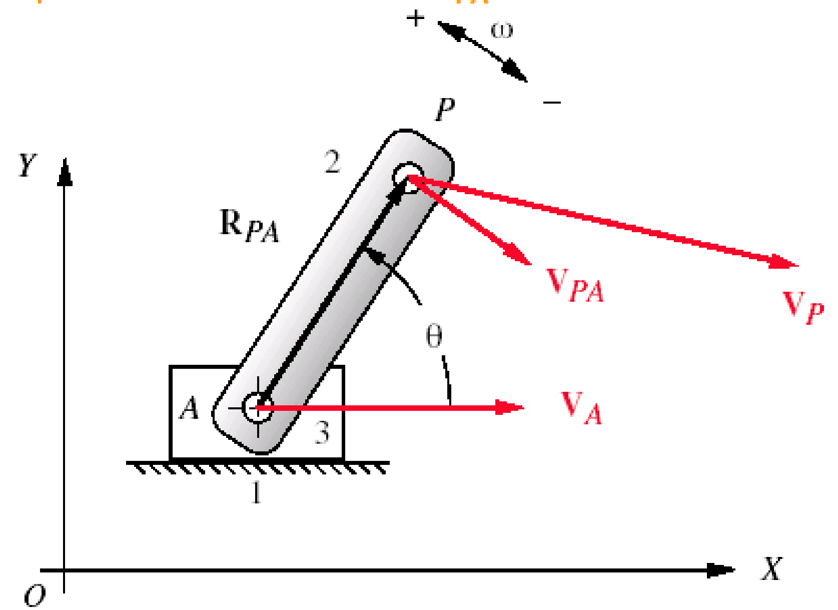
$$= (\omega_1 + \omega_2)\, r, \text{ if the links move in the opposite direction}$$

Velocity analysis: Velocity difference (2 points on the same body)

Reference: Pivot is no longer the origian of GCS, instead has a linear velocity

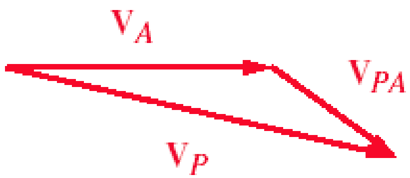


V_P is not the same as V_{PA}



$$V_{PA} = V_P - V_A$$

$$V_P = V_A + V_{PA}$$

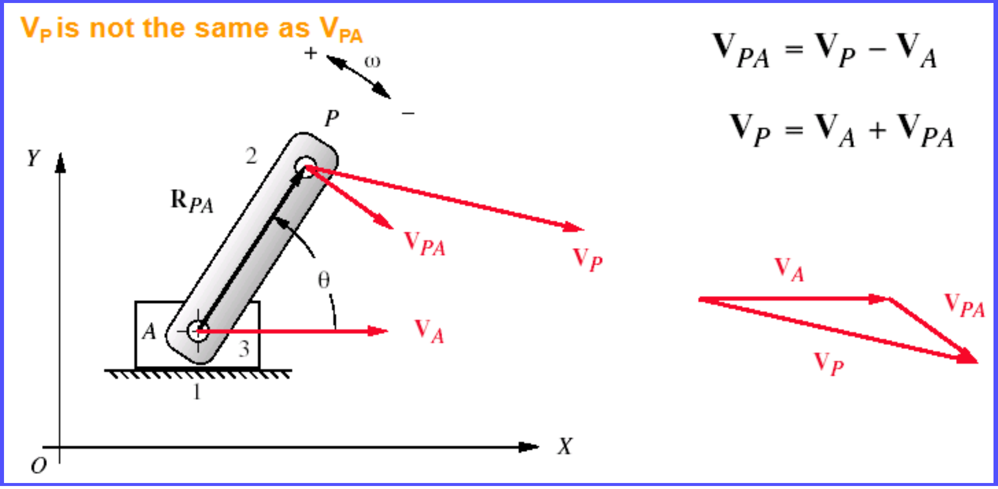


CASE 1:	*Two points in the same body* =>	velocity difference
CASE 2:	*Two points in different bodies* =>	relative velocity

→ $V_{PA} = V_P - V_A$

Velocity analysis: Relative Velocity (2 points on different bodies)

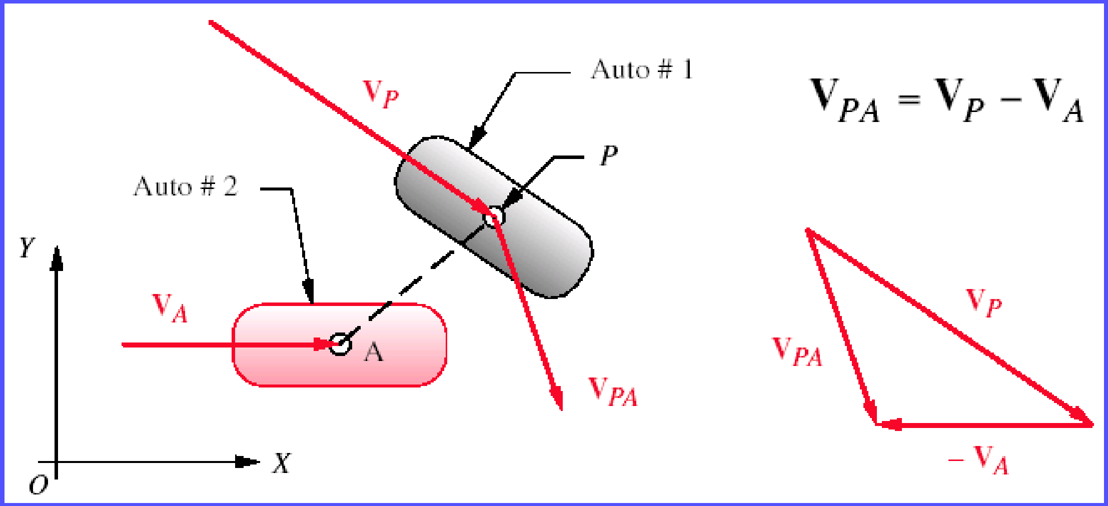
Reference: Pivot is no longer the origian of GCS, instead has a linear velocity



V_{PA} as the Velocity difference helps find the resultant.

CASE 1:	*Two points in the same body* =>	velocity difference
CASE 2:	*Two points in different bodies* =>	relative velocity

$$V_{PA} = V_P - V_A$$

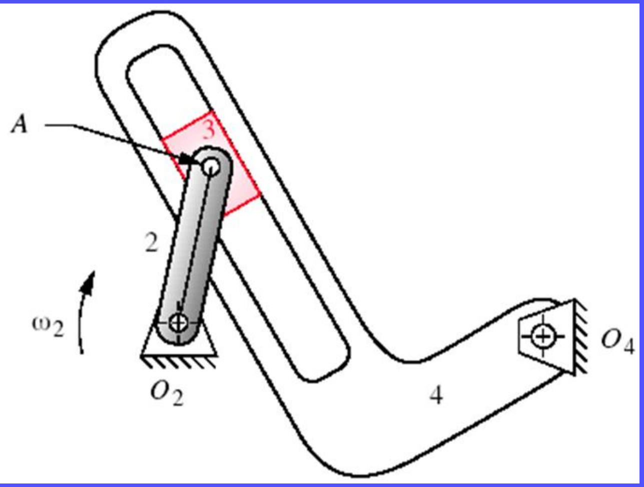


V_{PA} as the relative velocity is the resultant.

V_{PA} is not perpendicular to the line joining P and A.

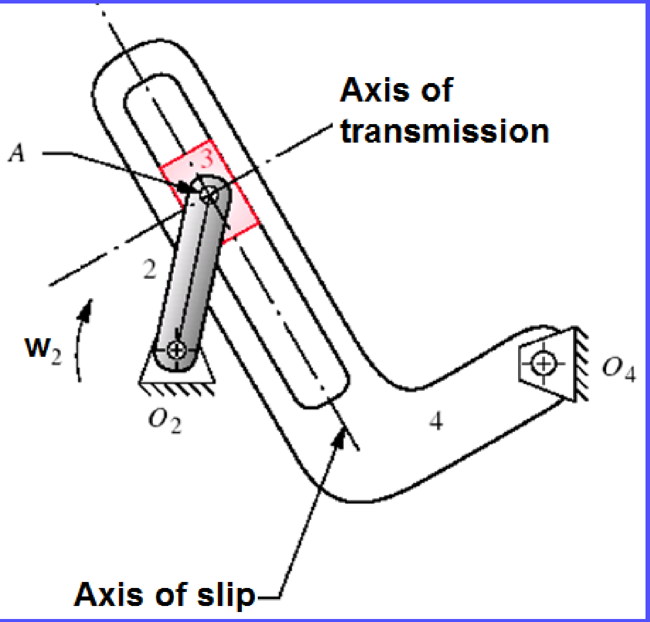
When P and A are not on the same body, the resultant vector Is different.

Velocity analysis: Velocity of slip



Both the links forming the sliding joint, are not grounded, implying a floating sliding joint.

Notably, point A belongs to two different bodies, namely, 2 &3, Implying case2: relative velocity.

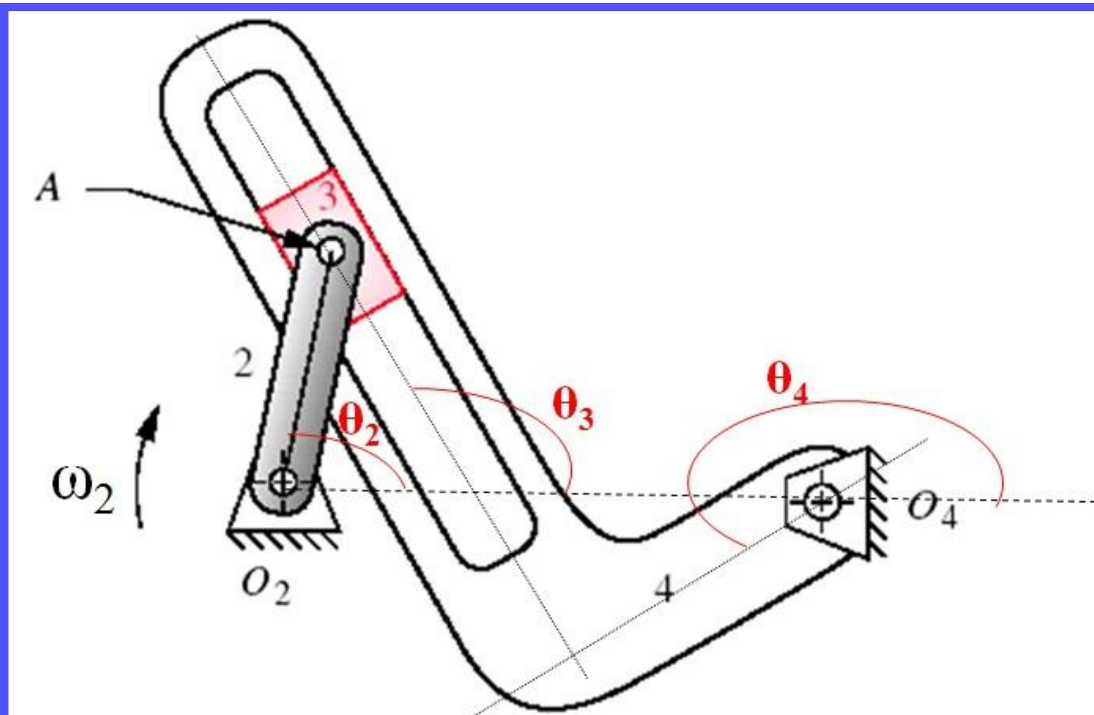


Axis of Slip: Line along which sliding occurs between links 3 & 4.

Axis of transmission: The line along which we can transmit motion or force across the slider joint (except friction - assumed negligible)

Velocity analysis: Velocity of slip

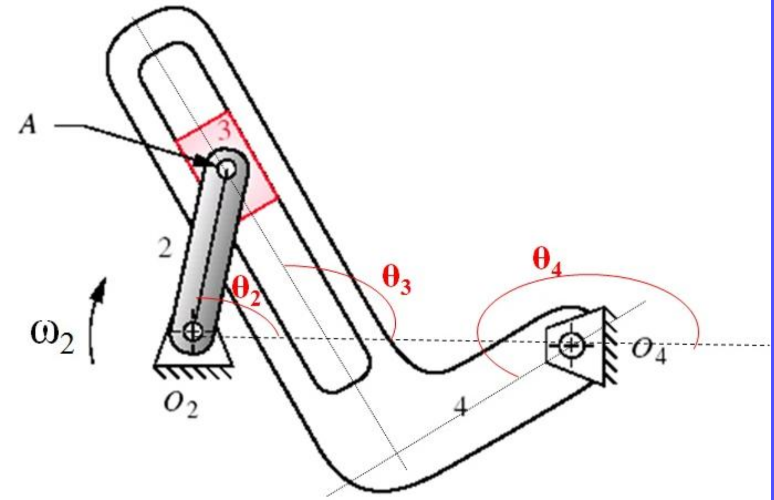
Problem: Given θ_2, θ_3, θ_4, ω_2, find ω_3, ω_4, $\mathbf{V}_A$, by graphical methods.

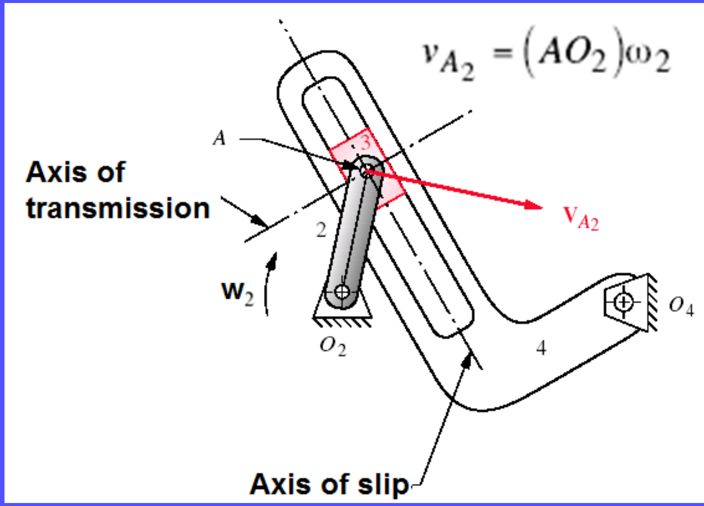


Intuition: The axis of 3 & 4 have a fixed geometric relationship, hence the rate of change of θ_3 θ_4 will remain the same: $\omega_3=\omega_4$

$$V_{A3}=V_{A2}$$

Velocity analysis: Velocity of slip





1	Draw the **axis of slip** and **axis of transmission** through point A.

2 Start at the end of the linkage for which you have the most information. Calculate the magnitude of the velocity of **point A as part of link 2** (A_2)

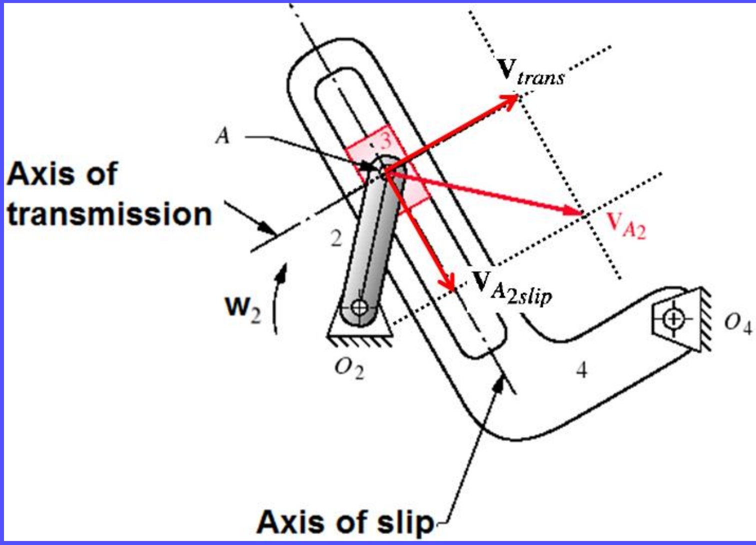
$$v_{A_2} = (AO_2)\omega_2$$

Perpendicular to O_2A

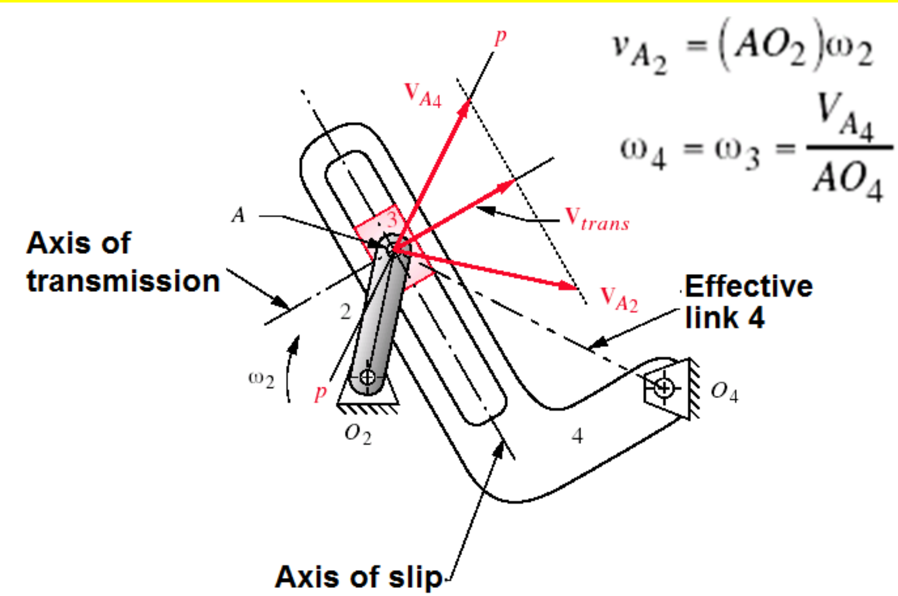
Sense as ω_2

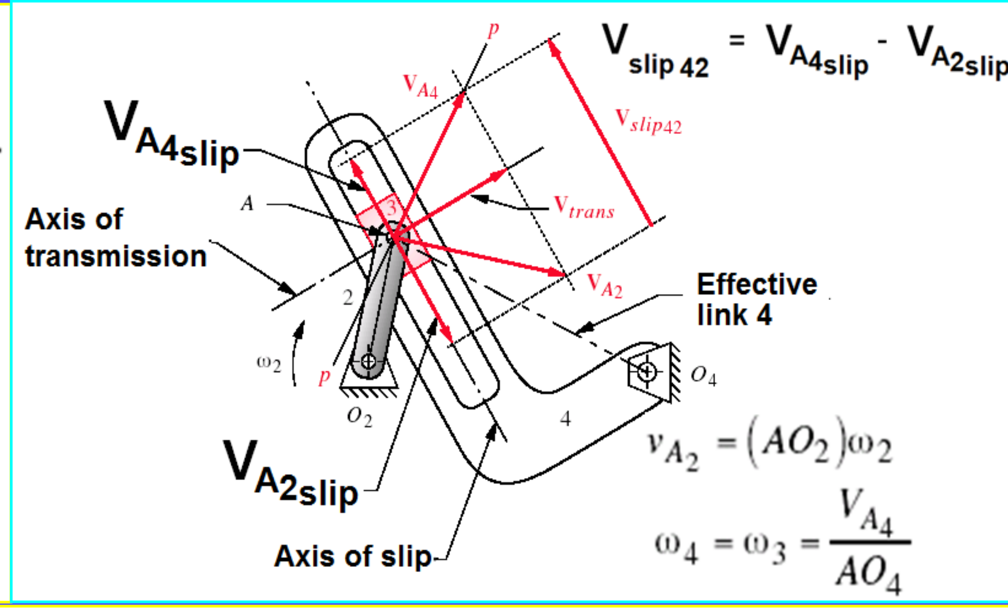
3 Project $\mathbf{V}_{A_2}$ onto the axis of slip and the axis of transmission to create the components $\mathbf{V}_{A_{2slip}}$ and $\mathbf{V}_{trans}$ of $\mathbf{V}_{A_2}$, respectively.

4 Note that link 3 is pin-jointed to link 2, so $\mathbf{V}_{A_3} = \mathbf{V}_{A_2}$.



Velocity analysis: Velocity of slip





5 Note that the direction of the velocity of point $\mathbf{V}_{A_4}$ is predictable since all points on link 4 are pivoting in pure rotation about point O_4. Draw the line *pp* through point *A* and perpendicular to the effective link 4, AO_4. Line *pp* is the direction of velocity $\mathbf{V}_{A_4}$.

6 Construct the true magnitude of velocity vector $\mathbf{V}_{A_4}$ by extending the projection of the **transmission component** $\mathbf{V}_{trans}$ until it intersects line *pp*.

7 Project $\mathbf{V}_{A_4}$ onto the axis of slip to create the **slip component** $\mathbf{V}_{A_{4slip}}$.

$$V_{slip_{42}} = V_{A_{4slip}} - V_{A_{2slip}}$$

8 Write the relative velocity vector equation for the **slip components** of point A_2 ver. point A_4.

9 The angular velocities of links 3 and 4 are identical because they share the slider joint and must rotate together.

$$\omega_4 = \omega_3 = \frac{V_{A_4}}{AO_4}$$

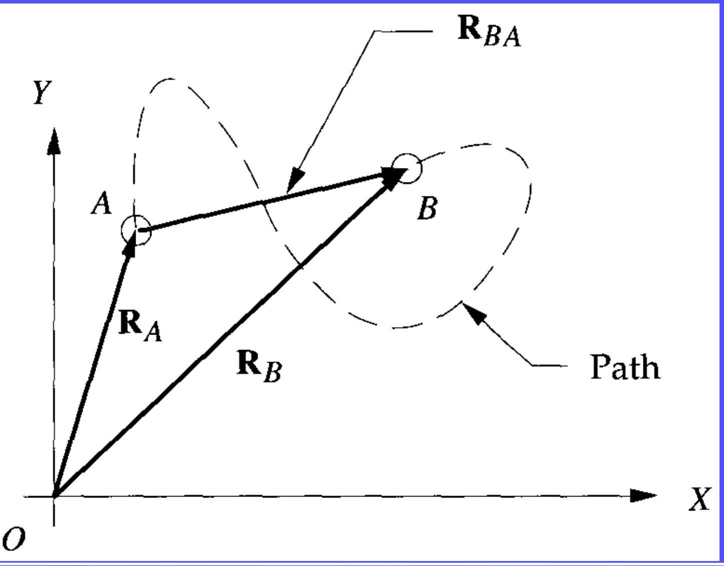
Velocity analysis:

Vector loop equations

&

Complex number notation

Velocity analysis: Refreshing basics from position analysis

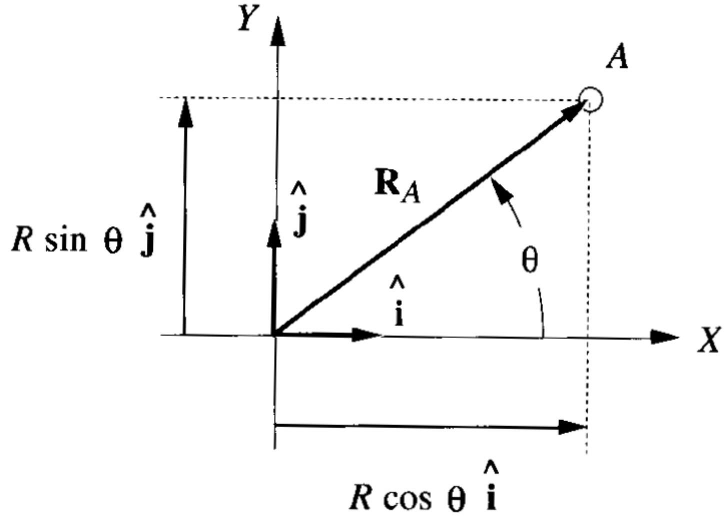


The position of B with respect to A = Absolute position of B minus that of A; (absolute implying the origin of the GCS.)

$$R_{BA} = R_{BO} - R_{AO} = R_B - R_A$$

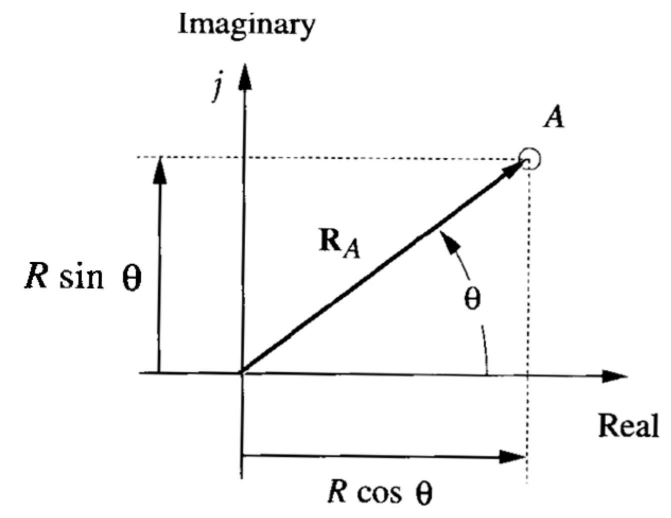
1: *One body in two successive positions* => *position difference*

2: *Two bodies simult. in separate positions* => *relative position*



Polar form: $|\mathbf{R}_A| \; @ \angle \theta$

Cartesian form: $R\cos\theta\,\hat{\mathbf{i}}$, $R\sin\theta\,\hat{\mathbf{j}}$



Polar form: $R\,e^{j\theta}$

Euler identity: $e^{\pm j\theta} = \cos\theta \pm j\sin\theta$

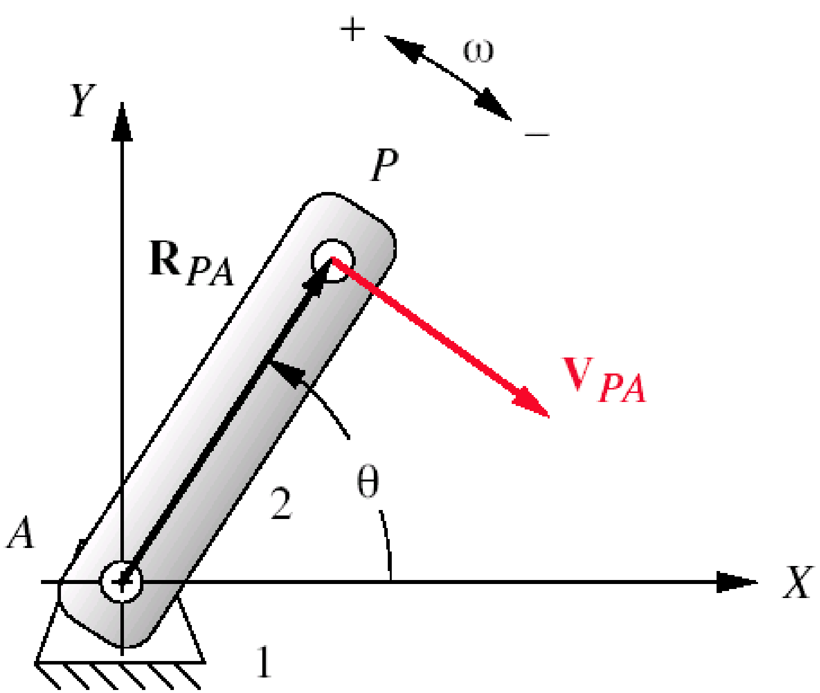
Velocity analysis: Complex number notation

Velocity is rate of change of position with respect to time.

- Position (R) is a vector quantity, so is velocity
- Velocity can be linear (V) or angular (θ)

Reference: Global Co-ordinate System (pivot: GCS origin)

$V_P = V_{PA}$



$$\mathbf{R}_{PA} = pe^{j\theta}$$

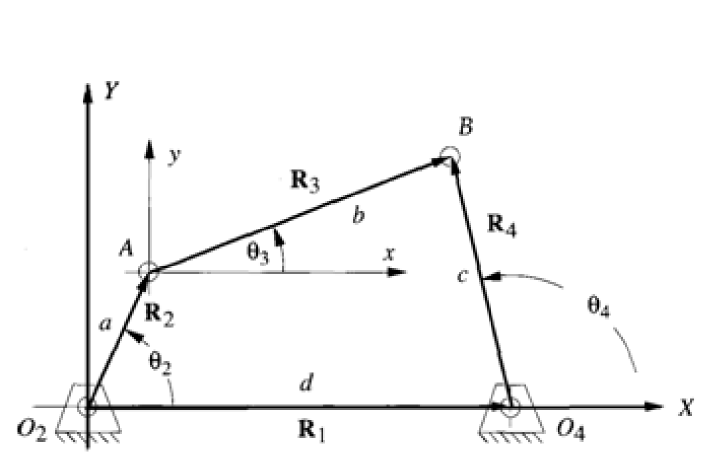
$$\mathbf{V} = \frac{d\mathbf{R}}{dt} \qquad \omega = \frac{d\theta}{dt};$$

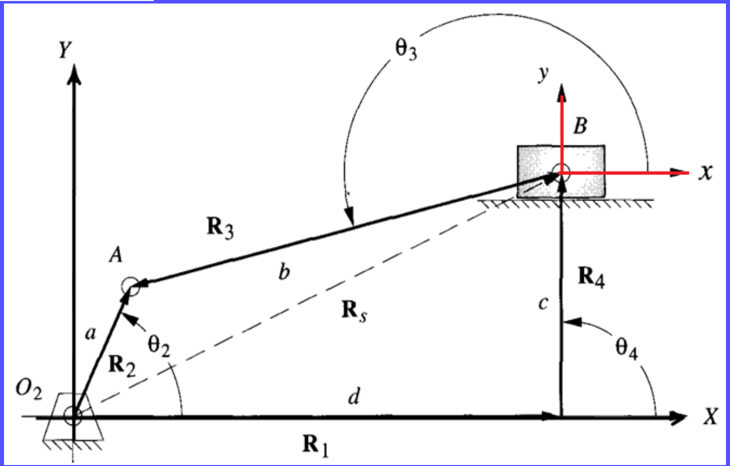
$$\mathbf{V}_{PA} = \frac{d\mathbf{R}_{PA}}{dt} = p\,je^{j\theta}\frac{d\theta}{dt}$$

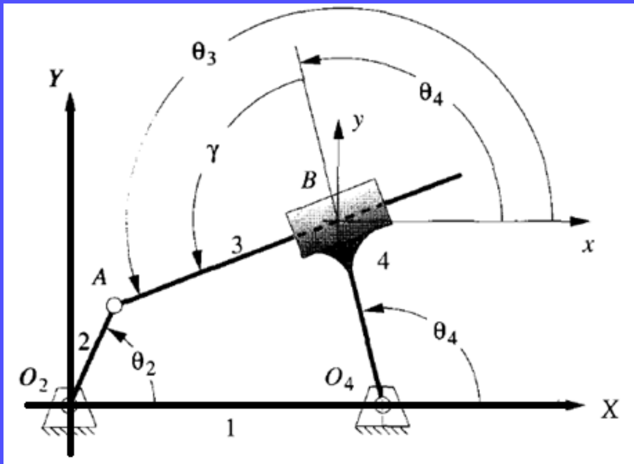
$$\mathbf{V}_{PA} = p\omega\, je^{j\theta} = j\,\omega\,\mathbf{R}_{PA}$$

The velocity vector is rotated through 90º w.r.t the original position vector, where the sense of the velocity vector is dictated by the sign of ω, where anticlockwise may be taken as positive

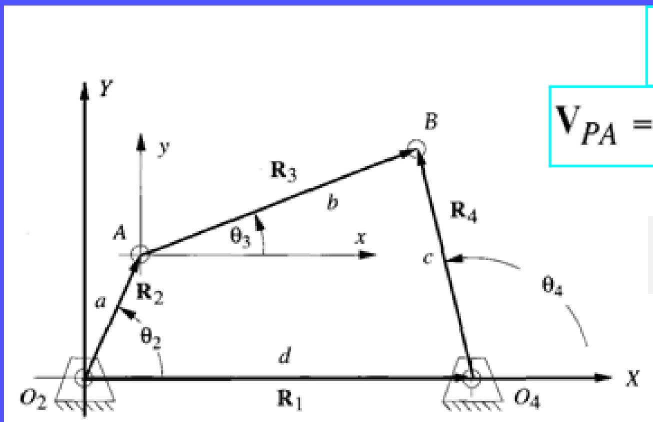
Velocity analysis: Vector loop equation







Velocity analysis: Vector loop equation- Problem 1



$$\mathbf{R}_{PA} = pe^{j\theta}$$

$$\mathbf{V}_{PA} = p\omega\, je^{j\theta} = j\,\omega\,\mathbf{R}_{PA}$$

θ_3, θ_4 ?

$$\mathbf{R}_2 + \mathbf{R}_3 - \mathbf{R}_4 - \mathbf{R}_1 = 0$$

$$ae^{j\theta_2} + be^{j\theta_3} - ce^{j\theta_4} - de^{j\theta_1} = 0$$

$$ja\omega_2 e^{j\theta_2} + jb\omega_3 e^{j\theta_3} - jc\omega_4 e^{j\theta_4} = 0$$

$$\mathbf{V}_A + \mathbf{V}_{BA} - \mathbf{V}_B = 0$$

$\theta_1$derivative = 0

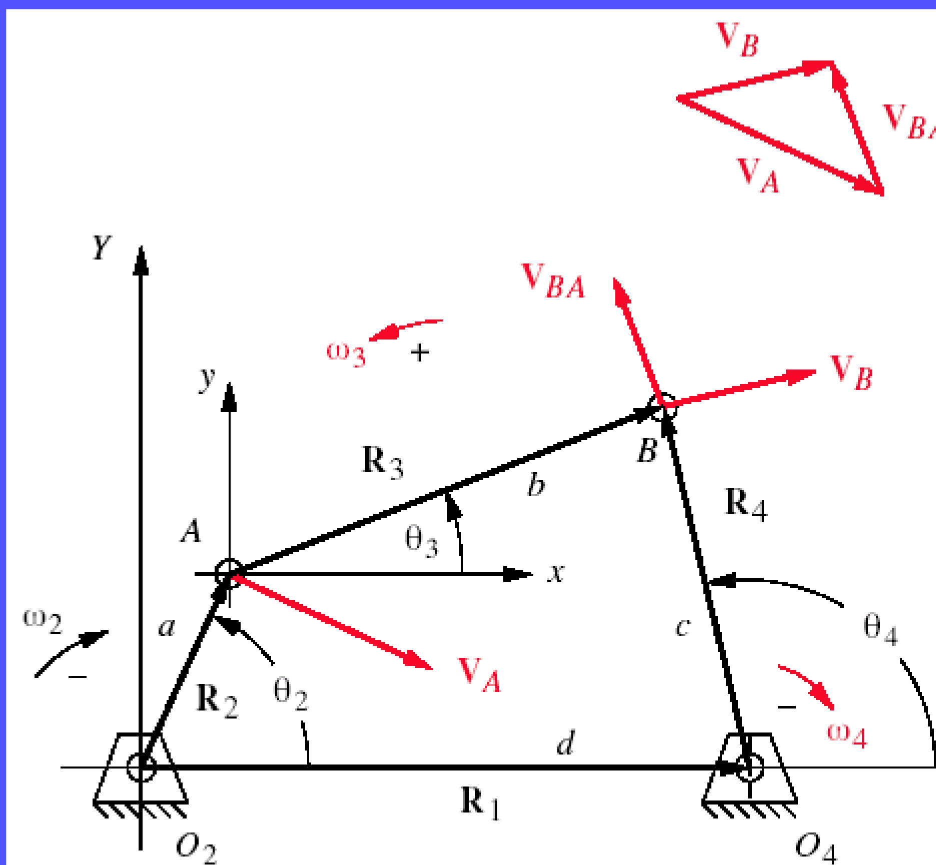
$$\mathbf{V}_A = ja\omega_2 e^{j\theta_2}$$
$$\mathbf{V}_{BA} = jb\omega_3 e^{j\theta_3}$$
$$\mathbf{V}_B = jc\omega_4 e^{j\theta_4}$$

$$a\omega_2(-\sin\theta_2 + j\cos\theta_2) + b\omega_3(-\sin\theta_3 + j\cos\theta_3)$$
$$-c\omega_4(-\sin\theta_4 + j\cos\theta_4) = 0$$

$$-a\omega_2\sin\theta_2 - b\omega_3\sin\theta_3 + c\omega_4\sin\theta_4 = 0$$

$$a\omega_2\cos\theta_2 + b\omega_3\cos\theta_3 - c\omega_4\cos\theta_4 = 0$$

$$\omega_3 = \frac{a\omega_2}{b}\frac{\sin(\theta_4-\theta_2)}{\sin(\theta_3-\theta_4)} \qquad \omega_4 = \frac{a\omega_2}{c}\frac{\sin(\theta_2-\theta_3)}{\sin(\theta_4-\theta_3)}$$

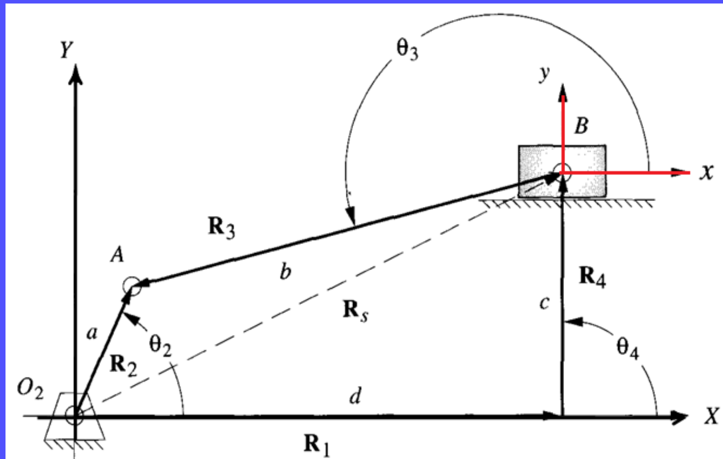


$$\mathbf{V}_A = ja\omega_2(\cos\theta_2 + j\sin\theta_2) = a\omega_2(-\sin\theta_2 + j\cos\theta_2)$$

$$\mathbf{V}_{BA} = jb\omega_3(\cos\theta_3 + j\sin\theta_3) = b\omega_3(-\sin\theta_3 + j\cos\theta_3)$$

$$\mathbf{V}_B = jc\omega_4(\cos\theta_4 + j\sin\theta_4) = c\omega_4(-\sin\theta_4 + j\cos\theta_4)$$

Velocity analysis: Vector loop equation- Problem 2



d, θ_3 ?

$$\mathbf{R}_2 - \mathbf{R}_3 - \mathbf{R}_4 - \mathbf{R}_1 = 0$$

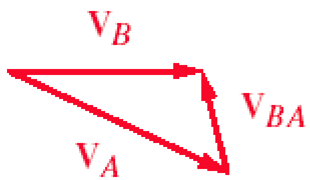
$$a e^{j\theta_2} - b e^{j\theta_3} - c e^{j\theta_4} - d e^{j\theta_1} = 0$$

$$\frac{d(d)}{dt} e^{j\theta_1} + d j e^{j\theta_1} \frac{d\theta_1}{dt}$$

$\frac{d\theta_1}{dt} = 0$ (θ_1 being a constant)

$e^{j\theta_1} = cos(\theta_1) + j sin(\theta_1) = 1$ for $\theta_1 = 0$

$$\frac{d(d)}{dt} e^{j\theta_1} + d j e^{j\theta_1} \frac{d\theta_1}{dt} = \frac{d(d)}{dt} = \dot{d}$$



$$\mathbf{R}_{PA} = p e^{j\theta}$$

$$\mathbf{V}_{PA} = p\omega\, j e^{j\theta} = j\,\omega\, \mathbf{R}_{PA}$$

$$j a \omega_2 e^{j\theta_2} - j b \omega_3 e^{j\theta_3} - \dot{d} = 0$$

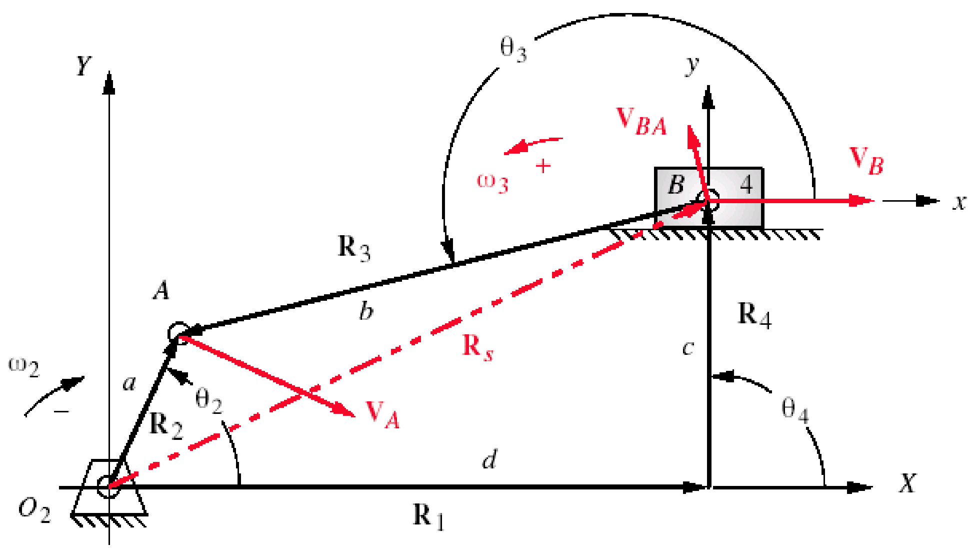
$$\mathbf{V}_A - \mathbf{V}_{AB} - \mathbf{V}_B = 0$$

$$\mathbf{V}_{BA} = -\mathbf{V}_{AB}$$

$$\mathbf{V}_B = \mathbf{V}_A + \mathbf{V}_{BA}$$

$$\mathbf{V}_A = a\omega_2(-\sin\theta_2 + j\cos\theta_2)$$

$$\mathbf{V}_{AB} = b\omega_3(-\sin\theta_3 + j\cos\theta_3)$$

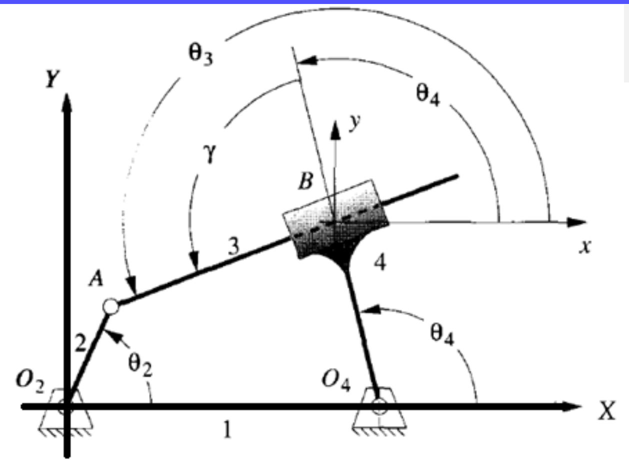


$$-a\omega_2 \sin\theta_2 + b\omega_3 \sin\theta_3 - \dot{d} = 0$$

$$a\omega_2 \cos\theta_2 - b\omega_3 \cos\theta_3 = 0$$

$$\omega_3 = \frac{a \cos\theta_2}{b \cos\theta_3}\omega_2 \qquad \dot{d} = -a\omega_2 \sin\theta_2 + b\omega_3 \sin\theta_3$$

Velocity analysis: Vector loop equation- Problem 3



b, θ_4 ?

$$\mathbf{R}_2 - \mathbf{R}_3 - \mathbf{R}_4 - \mathbf{R}_1 = 0$$

$$a e^{j\theta_2} - b e^{j\theta_3} - c e^{j\theta_4} - d e^{j\theta_1} = 0$$

$$ja\omega_2 e^{j\theta_2} - jb\omega_3 e^{j\theta_3} - \dot{b} e^{j\theta_3} - jc\omega_4 e^{j\theta_4} = 0$$

$\theta_3 = \theta_4 \pm \gamma$ implies $\omega_3 = \omega_4$

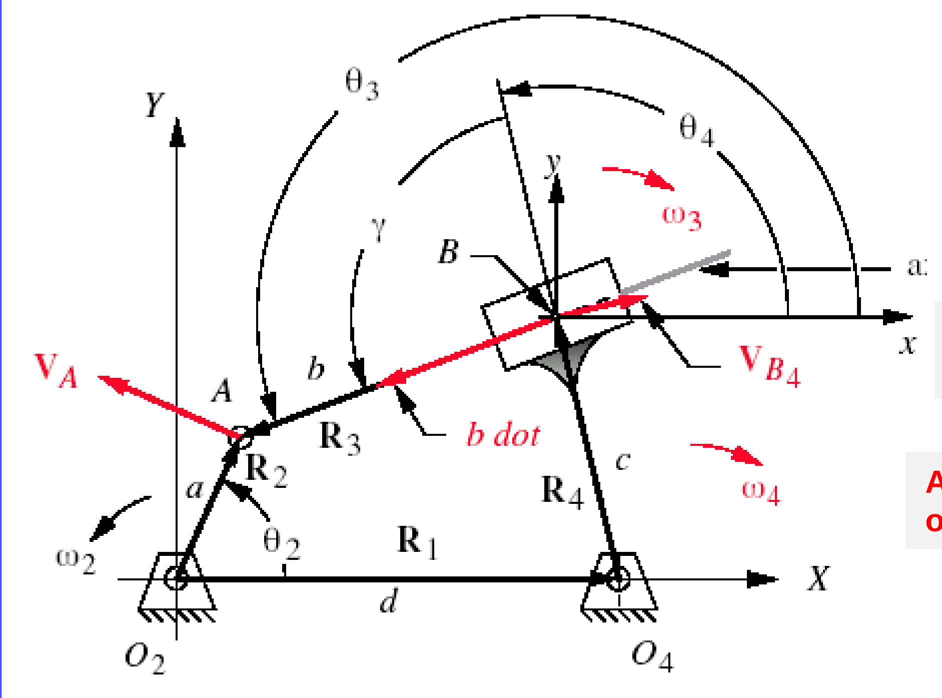
$$a\omega_2(-\sin\theta_2 + j\cos\theta_2) - b\omega_4(-\sin\theta_3 + j\cos\theta_3)$$
$$-\dot{b}(\cos\theta_3 + j\sin\theta_3) - c\omega_4(-\sin\theta_4 + j\cos\theta_4) = 0$$

$$-a\omega_2 \sin\theta_2 + b\omega_4 \sin\theta_3 - \dot{b}\cos\theta_3 + c\omega_4 \sin\theta_4 = 0$$
$$a\omega_2 \cos\theta_2 - b\omega_4 \cos\theta_3 - \dot{b}\sin\theta_3 - c\omega_4 \cos\theta_4 = 0$$

$$\dot{b}\cos\theta_3 = -a\omega_2 \sin\theta_2 + \omega_4(b\sin\theta_3 + c\sin\theta_4)$$
$$\dot{b}\sin\theta_3 = a\omega_2 \cos\theta_2 - \omega_4(b\cos\theta_3 + c\cos\theta_4)$$

$\theta_3 = \theta_4 \pm \gamma$

$\omega_3 = \omega_4$



Velocity of slip at point B

$$\dot{b} = \frac{-a\omega_2 \sin\theta_2 + \omega_4(b\sin\theta_3 + c\sin\theta_4)}{\cos\theta_3}$$

Angular Velocity of link 4

$$\omega_4 = \frac{a\omega_2 \cos(\theta_2 - \theta_3)}{b + c\cos\gamma}$$

$$\mathbf{V}_A = a\omega_2(-\sin\theta_2 + j\cos\theta_2)$$

$$\mathbf{V}_{B_4} = jc\omega_4 e^{j\theta_4} = c\omega_4(-\sin\theta_4 + j\cos\theta_4)$$