Common beverage can. Along with food containers, these are the most common pressure vessels. (AP/Wide World Photos)

Chapter 10: Stresses and Deformations in Cylinders

In all things, success depends on previous preparation. And without such preparation there is sure to be failure.

Confucius, *Analects*

Classes of Fit

Class	Type	Applications
1 (Loose)	Clearance	Where accuracy is not essential, such as in building and mining equipment
2 (Free)	Clearance	In rotating journals with speeds of 600 rpm or greater, such as in engines and some automotive parts
3 (Medium)	Clearance	In rotating journals with speeds under 600 rpm, such as in precision machine tools and precise automotive parts
4 (Snug)	Clearance	Where small clearance is permissible and where mating parts are not intended to move freely under load
5 (Wringing)	Interference	Where light tapping with a hammer is necessary to assemble the parts
6 (Tight)	Interference	In semipermanent assemblies suitable for drive or shrink fits on light sections
7 (Medium)	Interference	Where considerable pressure is required for assembly and for shrink fits of medium sections; suitable for press fits on generator and motor armatures and for automotive wheels
8 (Shrink)	Interference	Where considerable bonding between surfaces is required, such as locomotive wheels and heavy crankshaft disks of large engines

Table 10.1: Classes of fit.

Recommended Tolerances

Class	Allowance, a	Interference, δ	Hub tolerance, t_{lh}	Shaft tolerance, t_{ls}
1	$0.0025d^{2/3}$	—	$0.0025d^{1/3}$	$0.0025d^{1/3}$
2	$0.0014d^{2/3}$	—	$0.0013d^{1/3}$	$0.0013d^{1/3}$
3	$0.0009d^{2/3}$	—	$0.0008d^{1/3}$	$0.0008d^{1/3}$
4	0.000	—	$0.0006d^{1/3}$	$0.0004d^{1/3}$
5	—	0.000	$0.0006d^{1/3}$	$0.0004d^{1/3}$
6	—	$0.00025d$	$0.0006d^{1/3}$	$0.0006d^{1/3}$
7	—	$0.0005d$	$0.0006d^{1/3}$	$0.0006d^{1/3}$
8	—	$0.0010d$	$0.0006d^{1/3}$	$0.0006d^{1/3}$

Table 10.2: Recommended tolerances in *inches* for classes of fit.

Recommended Tolerances

Class	Allowance, a	Interference, δ	Hub tolerance, t_{lh}	Shaft tolerance, t_{ls}
1	$0.0073d^{2/3}$	—	$0.0216d^{1/3}$	$0.0216d^{1/3}$
2	$0.0041d^{2/3}$	—	$0.0112d^{1/3}$	$0.0112d^{1/3}$
3	$0.0026d^{2/3}$	—	$0.0069d^{1/3}$	$0.0069d^{1/3}$
4	0.000	—	$0.0052d^{1/3}$	$0.0035d^{1/3}$
5	—	0.000	$0.0052d^{1/3}$	$0.0035d^{1/3}$
6	—	$0.00025d$	$0.0052d^{1/3}$	$0.0052d^{1/3}$
7	—	$0.0005d$	$0.0052d^{1/3}$	$0.0052d^{1/3}$
8	—	$0.0010d$	$0.0052d^{1/3}$	$0.0052d^{1/3}$

Table 10.3: Recommended tolerances in *millimeters* for classes of fit.

Recommended Shaft and Hub Diameters

Type of fit	Hub diameter Maximum, $d_{h,\max}$	Hub diameter Minimum $d_{h,\min}$	Shaft diameter Maximum $d_{s,\max}$	Shaft diameter Minimum $d_{s,\min}$
Clearance	$d + t_{lh}$	d	$d - a$	$d - a - t_{ls}$
Interference	$d + t_{lh}$	d	$d + \delta + t_{ls}$	$d + \delta$

Table 10.4: Maximum and minimum diameters of shaft and hub for two types of fit.}

Thin-walled Pressure Vessel

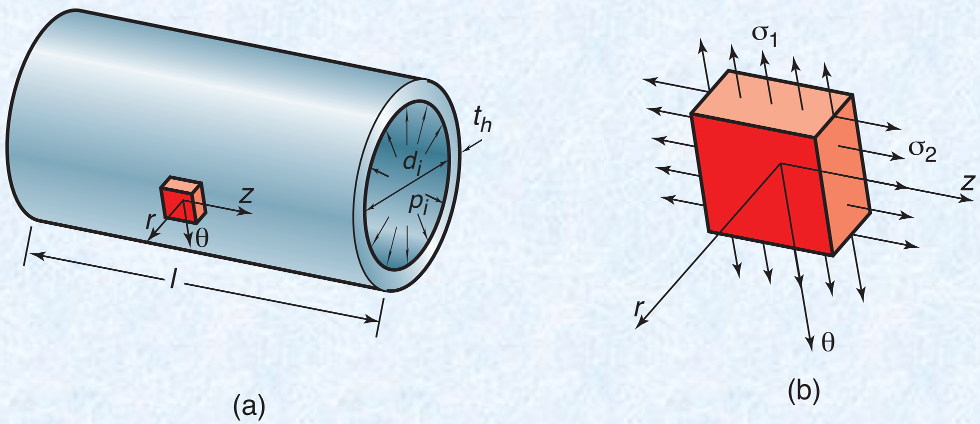

Figure 10.1: Internally pressurized thin-walled cylinder. (a) Stress element on cylinder; (b) stresses acting on element.

Criterion for thin vs. thick wall:

$$\frac{d_i}{t_h} > 40$$

Stresses for thin-walled cylinder:

$$\sigma_\theta = \frac{p_i r}{t_h}$$

$$\sigma_z = \frac{p_i r}{2t_h}$$

$$\sigma_r = p$$

Internally Pressurized Cylinder

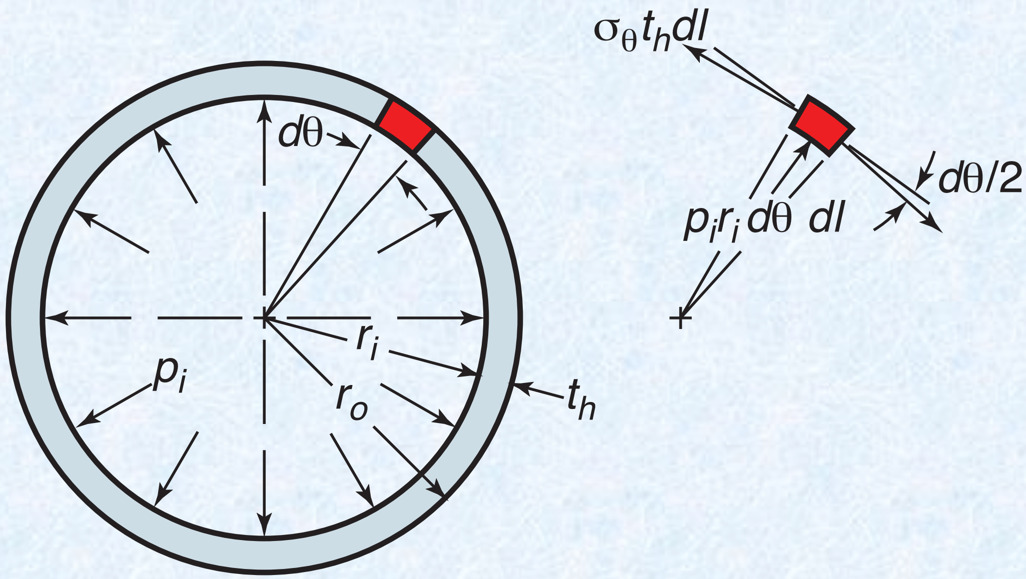

Figure 10.2: Front view of internally pressurized, thin-walled cylinder.

Pressurized Cylinder

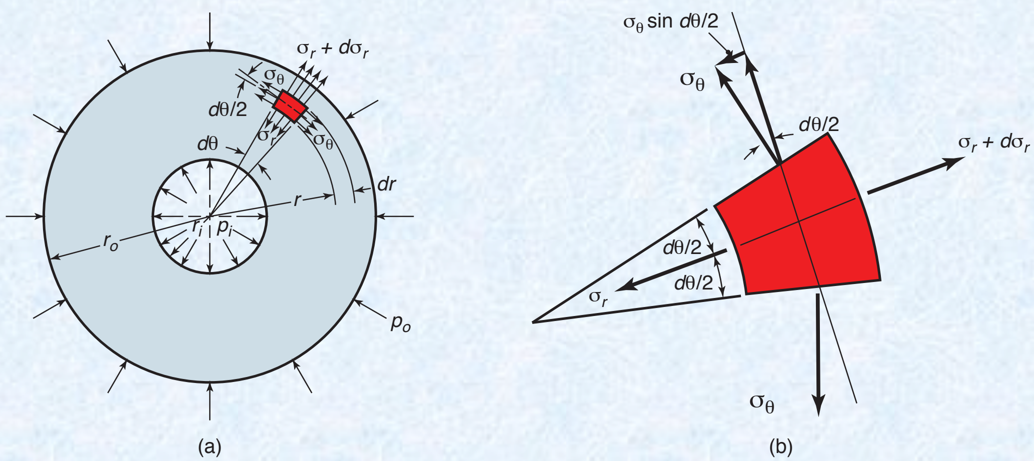

Figure 10.3: Complete front view of thick-walled cylinder internally and externally pressurized. (a) With stresses acting on cylinder; (b) detail of stresses acting on element.

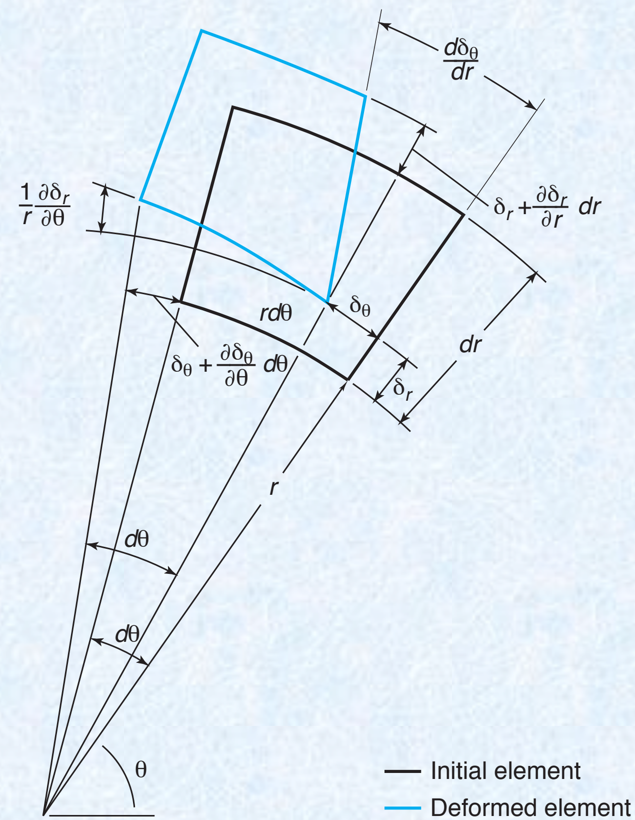

Figure 10.4: Cylindrical coordinate stress element before and after deformation.

Internally Pressurized Cylinder

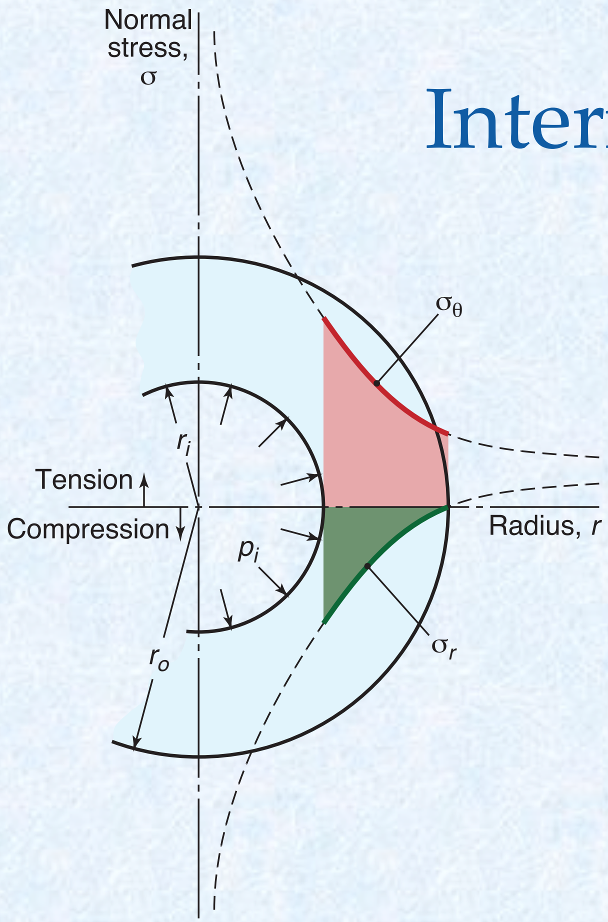

Figure 10.5: Internally pressurized, thick-walled cylinder showing circumferential (hoop) and radial stress for various radii.

Stress distribution:

$$\sigma_r = \frac{p_i r_i^2 \left(1 - \frac{r_o^2}{r^2}\right)}{r_o^2 - r_i^2}$$

$$\sigma_\theta = \frac{p_i r_i^2 \left(1 + \frac{r_o^2}{r^2}\right)}{r_o^2 - r_i^2}$$

Maxima occur at $r = r_i$:

$$\sigma_{r_{\max}} = -p_i$$

$$\sigma_{\theta_{\max}} = p_i \left(\frac{r_o^2 + r_i^2}{r_o^2 - r_i^2}\right)$$

Radial displacement:

$$\delta_r = \frac{p_i r_i}{E}\left(\frac{r_o^2 + r_i^2}{r_o^2 - r_i^2} + \nu\right)$$

Externally Pressurized Cylinder

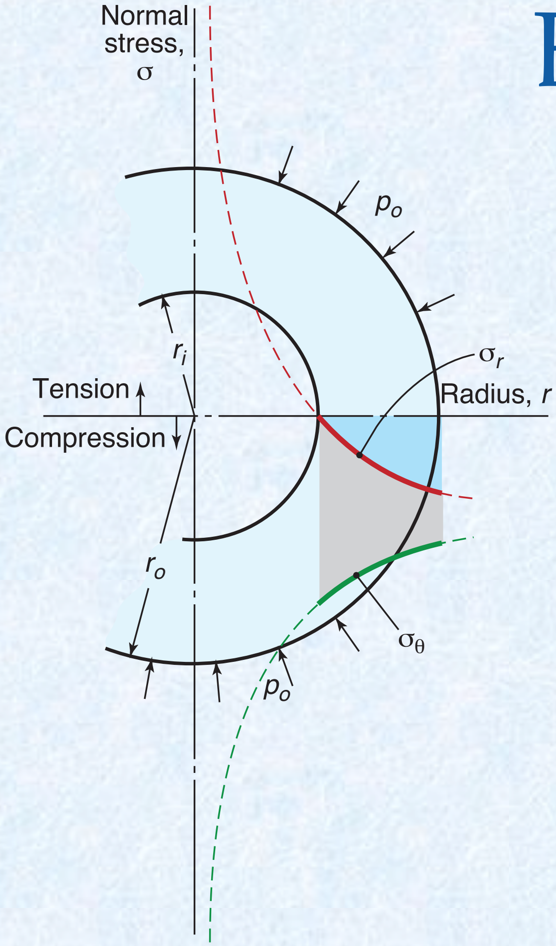

Figure 10.6: Externally pressurized, thick-walled cylinder showing circumferential (hoop) and radial stress for various radii.

Stress distribution:

$$\sigma_r = \frac{p_o r_o^2}{r_o^2 - r_i^2}\left(\frac{r_i^2}{r^2} - 1\right)$$

$$\sigma_\theta = -\frac{p_o r_o^2}{r_o^2 - r_i^2}\left(\frac{r_i^2}{r^2} + 1\right)$$

Maxima:

$$\sigma_{r,\max} = -p_o$$

$$\sigma_{\theta,\max} = -\frac{2r_o^2 p_o}{r_o^2 - r_i^2}$$

Design Procedure 10.1: Stress Analysis of Thick-Walled Cylinders

A common design problem is to determine the largest permissible external and/or internal pressure to which a cylinder can be subjected without failure. Axial stresses, if present, are negligibly small. The following design procedure is useful for such circumstances:

1. For internal pressurization, both the radial and circumferential stresses are largest at the inner radius. The von Mises stress for this plane stress case can be shown to be

$$\sigma_e = p_i \sqrt{\frac{3r_o^4 + r_i^4}{\left(r_o^2 - r_i^2\right)^2}}$$

   so that the allowable internal pressure is, from Eq.~(6.8), $p_i = \dfrac{S_y}{n_s} \dfrac{r_o^2 - r_i^2}{\sqrt{3r_o^4 + r_i^4}}$

2. For external pressurization, it can be shown that the larger von Mises stress occurs at the inner radius, with the stresses of $\sigma_r = 0$ and σ_θ given by Eq. (10.32). This yields an expression of allowable external pressure of:

$$p_o = \frac{S_y}{n_s} \frac{r_o^2 - r_i^2}{2r_o^2}$$

3. For combined internal and external pressurization, Eqs. (10.20) and (10.22) need to be substituted into a failure criterion from Ch. 6, such as the DET given for plane stress in Eqs. (6.10) and (6.11).

Rotating Cylinder

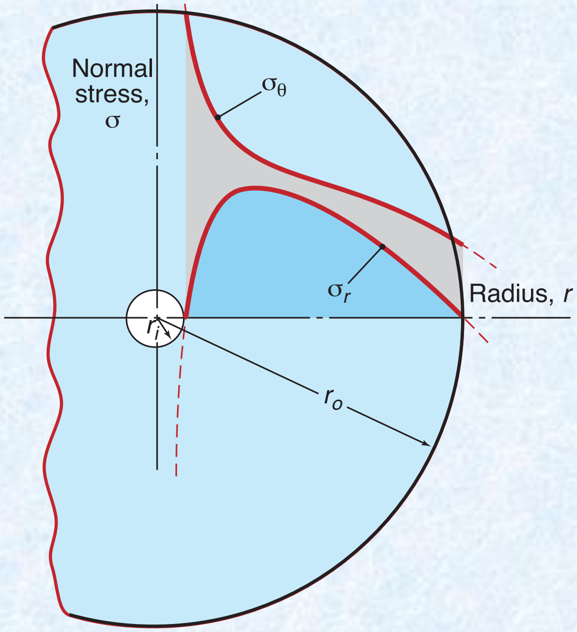

Figure 10.7: Stresses in rotating cylinder with central hole and no pressurization.

Stresses:

$$\sigma_\theta = \frac{3+\nu}{8}\rho\omega^2\left[r_i^2 + r_o^2 + \frac{r_i^2 r_o^2}{r^2} - \frac{1+3\nu}{3+\nu}r^2\right]$$

$$\sigma_r = \frac{3+\nu}{8}\rho\omega^2\left(r_i^2 + r_o^2 - \frac{r_i^2 r_o^2}{r^2} - r^2\right)$$

$$\sigma_{\theta,\max} = \frac{3+\nu}{4}\rho\omega^2\left[r_o^2 + \frac{r_i^2(1-\nu)}{3+\nu}\right]$$

$$\sigma_{r,\max} = \frac{3+\nu}{8}\rho\omega^2\left(r_i - r_o\right)^2$$

Rotating Solid Cylinder

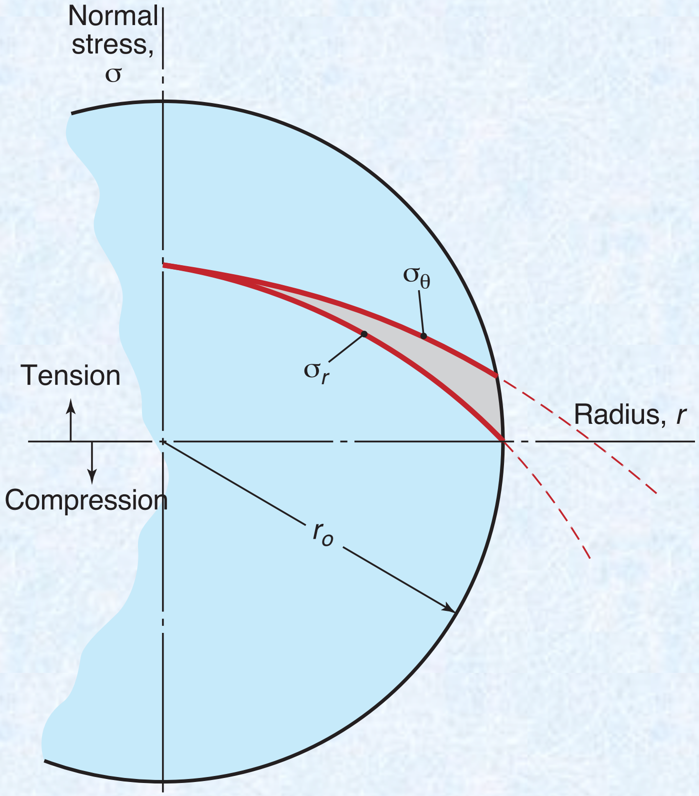

Figure 10.8: Stresses in rotating solid cylinder with no pressurization.

Stresses:

$$\sigma_\theta = \frac{3+\nu}{8}\rho\omega^2\left[r_o^2 - \frac{r^2(1+3\nu)}{3+\nu}\right]$$

$$\sigma_r = \frac{3+\nu}{8}\rho\omega^2\left(r_o^2 - r^2\right)$$

$$\sigma_{\theta,\max} = \sigma_{r,\max} = \frac{3+\nu}{8}\rho(r_o\omega)^2$$

Interference Fit

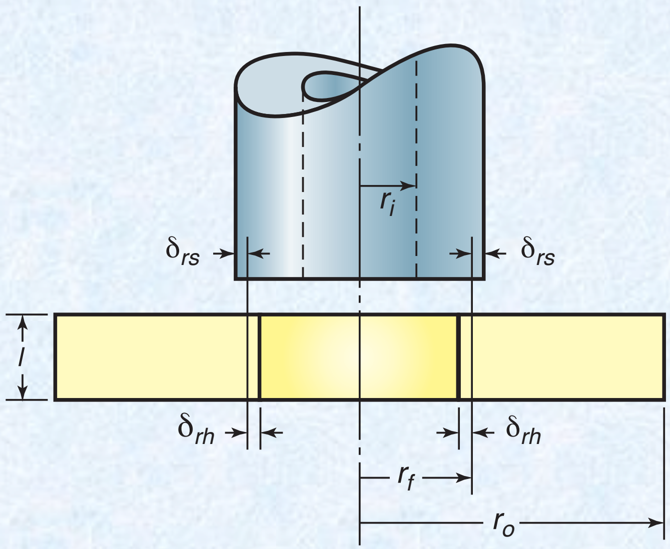

Figure 10.9: Side view showing interference in press fit of hollow shaft to hub.

For hub and shaft of the same material, and a solid shaft:

$$\delta_r = \frac{2 r_f p_f r_o^2}{E\left(r_o^2 - r_f^2\right)}$$

Torque that can be transmitted:

$$T = P_{\max} r_f = 2\pi \mu r_f^2 l p_f$$

Interference Fit

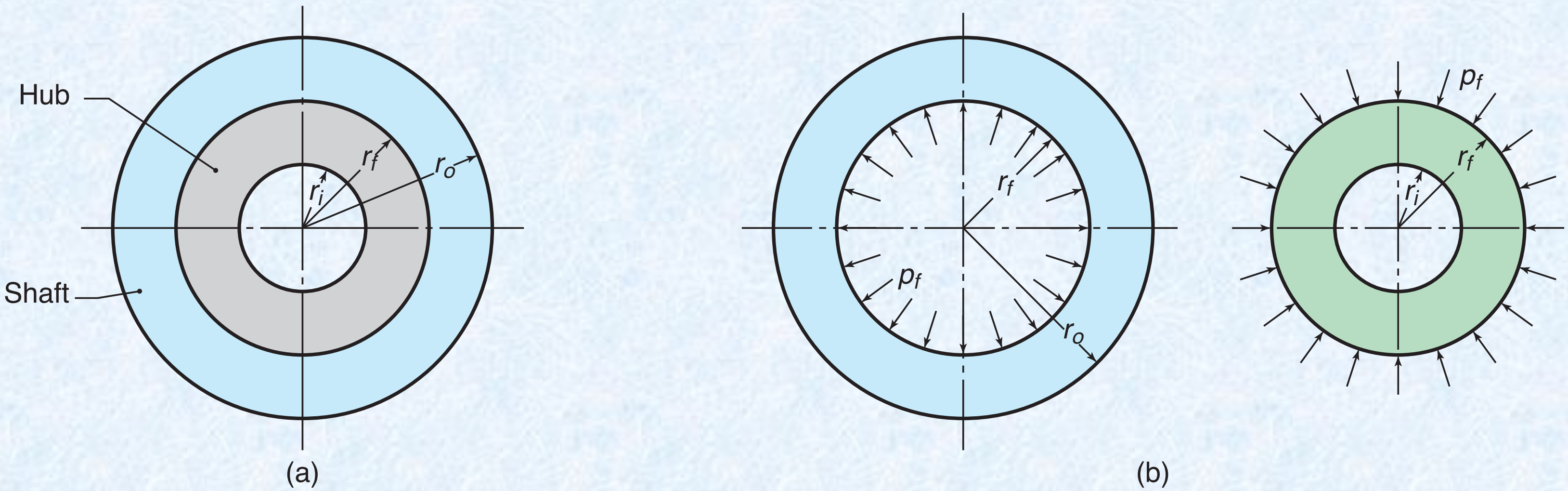

Figure 10.10: Front view showing (a) cylinder assembled with an interference fit and (b) hub and hollow shaft disassembled (also showing interference pressure).

Example 10.10

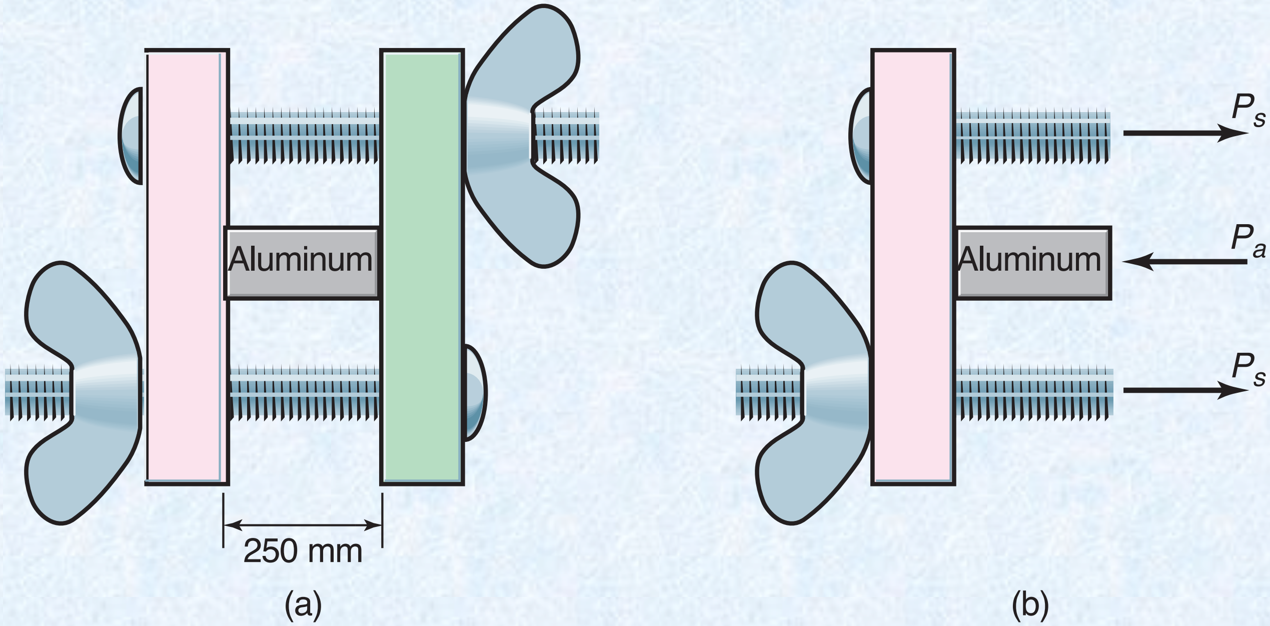

Figure 10.11: (a) Block placed between two rigid jaws of clamp, and (b) associated forces.

Die Casting Machine

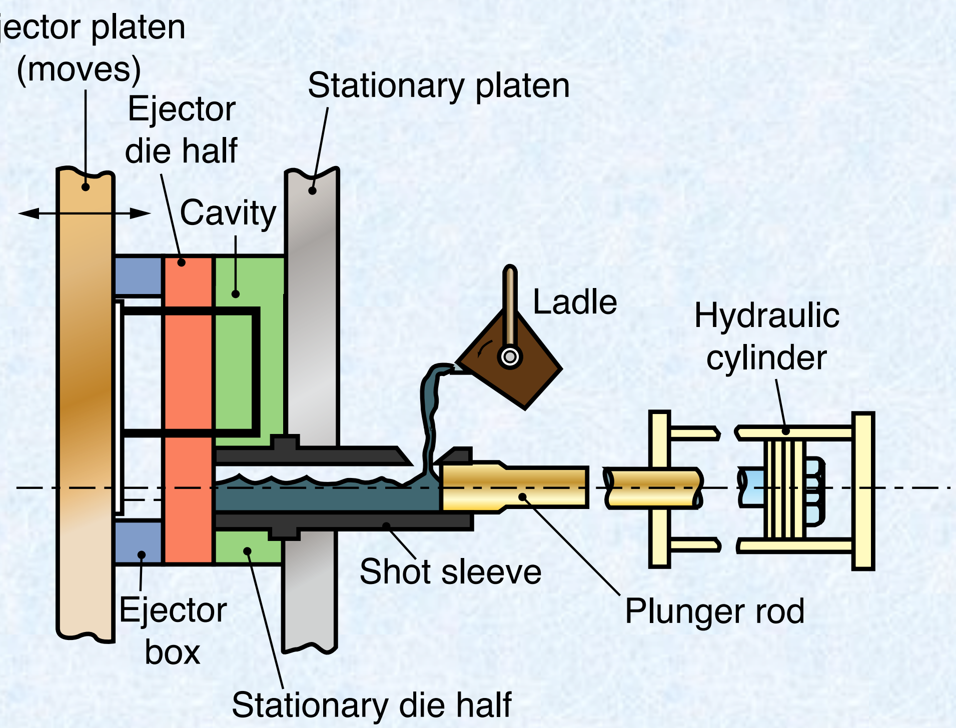

Figure 10.11: Schematic illustration of a die casting machine. *Source:* From Kalpakjian and Schmid [2010].